UNIVERSITY OF MUMBAI

Syllabus

for F. Y. B. Sc. / F. Y. B. A. Semester I & II (CBCS)

Program: B. Sc. / B. A.

Course: Mathematics

with effect from the academic year 2020-2021

F. Y. B. Sc. (CBCS) SEMESTER I

CALCULUS I				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 101	I	Real Number System	2	3
	II	Sequences in $\mathbb{R}$		
	III	First Order First Degree Differential Equations		
ALGEBRA I				
USMT 102	I	Integers and Divisibility	2	3
	II	Functions, Relations and Binary Operations		
	III	Polynomials		
PRACTICALS				
USMTP01	-	Practicals based on USMT101, USMT102	2	2

F. Y. B. A. (CBCS) SEMESTER I

CALCULUS I				
Course Code	UNIT	TOPICS	Credits	L/Week
UAMT 101	I	Real Number System	3	3
	II	Real Sequences		
	III	First Order First Degree Differential Equations		
Tutorials				
	-	Tutorials based on UAMT101		

F. Y. B. Sc. (CBCS) SEMESTER II

CALCULUS II				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 201	I	Limits and Continuity	2	3
	II	Differentiability of functions		
	III	Applications of Differentiability		
DISCRETE MATHEMATICS				
USMT 202	I	Preliminary Counting	2	3
	II	Advanced Counting		
	III	Permutations and Recurrence Relation		
PRACTICALS				
USMTP02	-	Practicals based on USMT201, USMT202	2	2

F. Y. B. A. (CBCS) SEMESTER II

CALCULUS II				
Course Code	UNIT	TOPICS	Credits	L/Week
	I	Limits and Continuity		
	II	Differentiability of functions		
UAMT 201	III	Applications of Differentiability	3	3
TUTORIALS				
	-	Tutorials based on UAMT201		

Revised Syllabus in Mathematics
Choice Based Credit System
F. Y. B. Sc. / B. A. 2020-2021

Preamble:

The University of Mumbai has brought into force the revised syllabi as per the Choice Based Credit System (CBCS) for the First year B. Sc/ B. A. Programme in Mathematics from the academic year 2020-2021.

Mathematics has been fundamental to the development of science and technology. In recent decades, the extent of application of Mathematics to real world problems has increased by leaps and bounds. Taking into consideration the rapid changes in science and technology and new approaches in different areas of mathematics and related subjects like Physics, Statistics and Computer Sciences, the board of studies in Mathematics with concern of teachers of Mathematics from different colleges affiliated to University of Mumbai has prepared the syllabus of F.Y.B. Sc. / F. Y. B. A. Mathematics. The present syllabi of F. Y. B. Sc. for Semester I and Semester II has been designed as per U. G. C. Model curriculum so that the students learn Mathematics needed for these branches, learn basic concepts of Mathematics and are exposed to rigorous methods gently and slowly. The syllabi of F. Y. B. Sc. / F. Y. B. A. would consist of two semesters and each semester would comprise of two courses for F. Y. B. Sc. Mathematics and one course for each semester for F. Y. B. A. Mathematics. Course I is 'Calculus I and Calculus II'. Calculus is applied and needed in every conceivable branch of science. Course II, 'Algebra I and Discrete Mathematics' develops mathematical reasoning and logical thinking and has applications in science and technology.

Aims:

(1) Give the students a sufficient knowledge of fundamental principles, methods and a clear perception of innumerous power of mathematical ideas and tools and know how to use them by modeling, solving and interpreting.

(2) Reflecting the broad nature of the subject and developing mathematical tools for continuing further study in various fields of science.

(3) Enhancing students' overall development and to equip them with mathematical modeling abilities, problem solving skills, creative talent and power of communication necessary for various kinds of employment.

(4) A student should get adequate exposure to global and local concerns that explore them many aspects of Mathematical Sciences

Course outcomes:

1. Calculus (Sem I & II): This course gives introduction to basic concepts of Analysis with rigor and prepares students to study further courses in Analysis. Formal proofs are given lot of emphasis in this course which also enhances understanding of the subject of Mathematics as a whole. The portion on first order, first degree differentials prepares learner to get solutions of so many kinds of problems in all subjects of Science and also prepares learner for further studies of differential equations and related fields.

2. Algebra I (Sem I) & Discrete Mathematics (Sem II): This course gives expositions to number systems (Natural Numbers & Integers), like divisibility and prime numbers and

their properties. These topics later find use in advanced subjects like cryptography and its uses in cyber security and such related fields.

Teaching Pattern for Semester I

[1.] Three lectures per week per course.

[2.] One Practical per week per batch for each of the courses USMT101, USMT 102 (the batches to be formed as prescribed by the University).

[3.] One Tutorial per week per batch for course UAMT101 (the batches to be formed as prescribed by the University).

Teaching Pattern for Semester II

[1.] Three lectures per week per course.

[2.] One Practical per week per batch for each of the courses USMT201, USMT 202. (the batches to be formed as prescribed by the University).

[3.] One Tutorial per week per batch for the course UAMT201 (the batches to be formed as prescribed by the University).

F.Y.B.Sc. / F.Y.B.A. Mathematics

SEMESTER I

USMT 101 / UAMT 101: CALCULUS I

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

Unit 1 : Real Number System (15 Lectures)

(1) Real number system $\mathbb{R}$ and order properties of $\mathbb{R}$, absolute value $|\ |$ and its properties.

(2) AM-GM inequality, Cauchy-Schwarz inequality, Intervals and neighbourhoods, interior points, limit point, Hausdorff property.

(3) Bounded sets, statements of l.u.b. axiom and its consequences, supremum and infimum, maximum and minimum, Archimedean property and its applications, density of rationals.

Unit II: Sequences in $\mathbb{R}$ (15 Lectures)

(1) Definition of a sequence and examples, Convergence of sequences, every convergent sequences is bounded. Limit of a convergent sequence and uniqueness of limit, Divergent sequences.

(2) Convergence of standard sequences like $\left(\dfrac{1}{1+na}\right)$ $\forall a > 0$, (b^n) $\forall b$, $0 < b < 1$, $(c^{\frac{1}{n}})$ $\forall c > 0$, & $(n^{\frac{1}{n}})$.

(3) Algebra of convergent sequences, sandwich theorem, monotone sequences, monotone convergence theorem and consequences as convergence of $\left(\left(1+\dfrac{1}{n}\right)^n\right)$.

(4) Definition of subsequence, subsequence of a convergent sequence is convergent and converges to the same limit, definition of a Cauchy sequences, every convergent sequences s a Cauchy sequence and converse.

Unit III: First order First degree Differential equations (15 Lectures)

Review of Definition of a differential equation, order, degree, ordinary differential equation and partial differential equation, linear and non linear ODE. Solution of homogeneous and non-homogeneous differential equations of first order and first degree. Notion of partial derivatives.

(1) Exact Equations: General solution of Exact equations of first order and first degree. Necessary and sufficient condition for $Mdx + Ndy = 0$ to be exact. Non-exact equations: Rules for finding integrating factors (without proof) for non exact equations, such as :

i) $\dfrac{1}{M\ x + N\ y}$ is an I.F. if $M\ x + N\ y \neq 0$ and $Mdx + Ndy = 0$ is homogeneous.

ii) $\dfrac{1}{M\ x - N\ y}$ is an I.F. if $M\ x - N\ y \neq 0$ and $Mdx + Ndy = 0$ is of the form $f_1(x,y)\ y\ dx + f_2(x,y)\ x\ dy = 0$.

iii) $e^{\int f(x)\ dx}$ (resp $e^{\int g(y)\ dy}$) is an I.F. if $N \neq 0$ (resp $M \neq 0$) and $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ $\left(\text{resp}\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)\right)$ is a function of x (resp y) alone, say $f(x)$ (resp $g(y)$).

iv) Linear and reducible linear equations of first order, finding solutions of first order differential equations of the type for applications to orthogonal trajectories, population growth, and finding the current at a given time.

(2) Reduction of order :

(i) If the differential equation does not contain only the original function y, that is equations of Type $F(x, y', y'') = 0$.

(ii) If the differential equation does not contain the independent variable x that is, equations of Type $F(y, y', y'') = 0$.

Reference Books:

1. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
2. K. G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
3. R. G. Bartle- D. R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 1994.
4. Sudhir Ghorpade and Balmohan Limaye, A course in Calculus and Real Analysis, Springer International Ltd, 2000.
5. G. F. Simmons, Differential Equations with Applications and Historical Notes, McGraw Hill, 1972.
6. E. A. Coddington , An Introduction to Ordinary Differential Equations.Prentice Hall, 1961.
7. W. E. Boyce, R. C. DiPrima, Elementary Differential Equations and Boundary Value Problems, Wiely, 2013.

Additional Reference Books

1. T. M. Apostol, Calculus Volume I, Wiley & Sons (Asia) Pte, Ltd.
2. Richard Courant-Fritz John, A Introduction to Calculus and Analysis, Volume I, Springer.
3. Ajit kumar and S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
4. James Stewart, Calculus, Third Edition, Brooks/ cole Publishing Company, 1994.
5. D. A. Murray, Introductory Course in Differential Equations, Longmans, Green and Co., 1897.
6. A. R. Forsyth, A Treatise on Differential Equations, MacMillan and Co.,1956.

ALGEBRA I
USMT 102

Prerequisite :
Set Theory: Set, subset, union and intersection of two sets, empty set, universal set, complement of a set, De Morgan's laws, Cartesian product of two sets, Relations, Permutations ${}^nP_r$ and Combinations ${}^nC_r$.
Complex numbers: Addition and multiplication of complex numbers, modulus, amplitude and conjugate of a complex number.
Unit I : Integers & Divisibility (15 Lectures)

(1) Statements of well-ordering property of non-negative integers, Principle of finite induction (first and second) as a consequence of Well-Ordering Principle.

(2) Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.m.) of two non zero integers, basic properties of g.c.d. such as existence and uniqueness of g.c.d. of two non zero integers $a\&b$ and that the g.c.d. can be expressed as $ma + nb$ for some $m, n \in \mathbb{Z}$, Euclidean algorithm.

(3) Primes, Euclid's lemma, Fundamental Theorem of arithmetic, The set of primes is infinite, there are arbitrarily large gaps between primes, there exists infinitely many primes of the form $4n - 1$ or of the form $6n - 1$.

(4) Congruence, definition and elementary properties, Results about linear congruence equations. Examples.

Unit II : Functions, Relations and Binary Operations (15 Lectures)

(1) Definition of relation and function, domain, co-domain and range of a function, composite functions, examples, Direct image $f(A)$ and inverse image $f^{-1}(B)$ for a function f, injective, surjective, bijective functions, Composite of injective, surjective, bijective functions when defined, invertible functions, bijective functions are invertible and conversely, examples of functions including constant, identity, projection, inclusion, Binary operation as a function, properties, examples.

(2) Equivalence relation, Equivalence classes, properties such as two equivalences classes are either identical or disjoint, Definition of partition, every partition gives an equivalence relation and vice versa.

(3) Congruence is an equivalence relation on $\mathbb{Z}$, Residue classes and partition of $\mathbb{Z}$, Addition modulo$n$, Multiplication modulo n, examples.

Unit III: Polynomials (15 Lectures)

(1) Definition of a polynomial, polynomials over F where $F = \mathbb{Q}, \mathbb{R}$ or $\mathbb{C}$, Algebra of polynomials, degree of polynomial, basic properties.

(2) Division algorithm in $F[X]$ (without proof), and g.c.d of two polynomials and its basic properties, Euclidean algorithm (proof of the above results may be given only in the case of $\mathbb{Q}[X]$ with a remark that the results as well as the proofs remain valid in the case of $\mathbb{R}[X]$ or $\mathbb{C}[X]$).

(3) Roots of a polynomial, relation between roots and coefficients, multiplicity of a root. Elementary consequences such as the following.

(i) Remainder theorem, Factor theorem.

(ii) A polynomial of degree n has at most n roots.

(iii) Complex and non-real roots of a polynomials in $\mathbb{R}[X]$ occur in conjugate pairs.

(Emphasis on examples and problems in polynomials with real coefficients).

(4) Necessary condition for a rational number $\frac{p}{q}$ to be a root of a polynomial with integer coefficients (viz. p divides the constant coefficient and q divides the leading coefficient), corollary for monic polynomials (viz. a rational root of monic polynomial with integer coefficients is necessarily an integer). Simple consequence such as the irrationality is necessarily of $\sqrt{p}$ for any prime number p. Irreducible polynomials in $\mathbb{Q}[x]$, Unique Factorisation Theorem. Examples.

Reference Books:

1. David M. Burton, Elementary Number Theory, Seventh Edition, McGraw Hill Education (India) Private Ltd.

2. 2. Norman L. Biggs, Discrete Mathematics, Revised Edition, Clarendon Press, Oxford 1989.

Additional Reference Books

1. I. Niven and S. Zuckerman, Introduction to the theory of numbers, Third Edition, Wiley Eastern, New Delhi, 1972.

2. G. Birkoff and S. Maclane, A Survey of Modern Algebra, Third Edition, Mac Millan, New York, 1965.

3. N. S. Gopalkrishnan, University Algebra, Ne Age International Ltd, Reprint 2013.

4. I .N. Herstein, Topics in Algebra, John Wiley, 2006.

5. P. B. Bhattacharya S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, New Age International, 1994.

6. Kenneth Rosen, Discrete Mathematics and its applications, Mc-Graw Hill, International Edition, Mathematics Series.

PRACTICALS FOR F.Y.B.Sc
USMTP01 – Practicals

A. **Practicals for USMT101/ UAMT 101:**

(1) Algebraic and Order Properties of Real Numbers and Inequalities

(2) Hausdorff Property and LUB Axiom of $\mathbb{R}$, Archimedian Property.

(3) Convergence and divergence of sequences, bounded sequences, Sandwich Theorem.

(4) Cauchy sequences, monotonic sequences, non-monotonic sequences.

(5) Solving exact and non-exact, linear, reducible to linear differential equations.

(6) Reduction of order of Differential Equations, Applications of Differential Equations.

(7) Miscellaneous Theoretical Questions based on full paper.

B. **Practicals for USMT102:**

(1) Mathematical induction ,Division Algorithm, Euclidean algorithm in $\mathbb{Z}$, Examples on expressing the gcd. of two non zero integers $a\&b$ as $ma + nb$ for some $m, n \in \mathbb{Z}$,

(2) Primes and the Fundamental theorem of Arithmetic, Euclid's lemma, there exists infinitely many primes of the form $4n - 1$ or of the form $6n - 1$.

(3) Functions, Bijective and Invertible functions, Compositions of functions.

(4) Binary Operation, Equivalence Relations, Partition and Equivalence classes.

(5) Polynomial (I)

(6) Polynomial (II)

(7) Miscellaneous Theoretical Questions based on full paper.

TUTORIALS FOR F.Y.B.A

Tutorials for UAMT101 :

(1) Algebraic and Order Properties of Real Numbers and Inequalities

(2) Hausdorff Property and LUB Axiom of $\mathbb{R}$, Archimedian Property.

(3) Convergence and divergence of sequences, bounded sequences, Sandwich Theorem.

(4) Cauchy sequences, monotonic sequences, non-monotonic sequences.

(5) Solving exact and non-exact, linear, reducible to linear differential equations.

(6) Reduction of order of Differential Equations, Applications of Differential Equations.

(7) Miscellaneous Theoretical Questions based on full paper.

Semester II
USMT 201 / UAMT201: CALCULUS II

Unit-I: Limits and Continuity (15 Lectures)

{Brief review: Domain and range of a function, injective function, surjective function, bijective function, composite of two functions (when defined), Inverse of a bijective function. Graphs of some standard functions such as $|x|$, e^x, $\log x$, ax^2+bx+c, $\frac{1}{x}$, $x^n\ n \geq 3)$, $\sin x$, $\cos x$, $\tan x$, $\sin\left(\frac{1}{x}\right)$, $x^2 \sin\left(\frac{1}{x}\right)$ over suitable intervals of $\mathbb{R}$. No direct questions to be added.}

(1) $\varepsilon - \delta$ definition of Limit of a function, uniqueness of limit if it exists, algebra of limits, limits of composite function, sandwich theorem, left-hand-limit $\lim_{x \longrightarrow a^-} f(x)$, right-hand-limit $\lim_{x \longrightarrow a^+} f(x)$, non-existence of limits, $\lim_{x \longrightarrow -\infty} f(x)$, $\lim_{x \longrightarrow \infty} f(x)$ and $\lim_{x \longrightarrow a} f(x) = \pm\infty$.

(2) Continuous functions: Continuity of a real valued function at a point and on a set using $\epsilon - \delta$ definition, examples, Continuity of a real valued function at end points of domain using $\epsilon - \delta$ definition, f is continuous at a if and only if $\lim_{x \longrightarrow a} f(x)$ exists and equals to $f(a)$, Sequential continuity, Algebra of continuous functions, discontinuous functions, examples of removable and essential discontinuity.

(3) Intermediate Value theorem and its applications, Bolzano-Weierstrass theorem (statement only): A continuous function on a closed and bounded interval is bounded and attains its bounds.

Unit-II: Differentiability of functions (15 Lectures)

(1) Differentiation of real valued function of one variable: Definition of differentiability of a function at a point of an open interval, examples of differentiable and non differentiable functions, differentiable functions are continuous but not conversely, algebra of differentiable functions.

(2) Chain rule, Higher order derivatives, Leibniz rule, Derivative of inverse functions, Implicit differentiation (only examples)

Unit-III: Applications of differentiability (15 Lectures)

(1) Rolle's Theorem, Lagrange's and Cauchy's Mean Value Theorems, applications and examples, Monotone increasing and decreasing functions, examples.

(2) L-Hospital rule (without proof), examples of indeterminate forms, Taylor's theorem with Lagrange's form of remainder with proof, Taylor polynomial and applications.

(3) Definition of critical point, local maximum/minimum, necessary condition, stationary points, second derivative test, examples, concave/convex functions, point of inflection.

(4) Sketching of graphs of functions using properties.

Reference books:

1. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
2. James Stewart, Calculus, Third Edition, Brooks/ Cole Publishing company, 1994.
3. T. M. Apostol, Calculus, Vol I, Wiley And Sons (Asia) Pte. Ltd.

4. Sudhir Ghorpade and Balmohan Limaye, A course in Calculus and Real Analysis, Springer International Ltd, 2000.

Additional Reference:

1. Richard Courant and Fritz John, A Introduction to Calculus and Analysis, Volume-I, Springer.
2. Ajit Kumar and S. Kumaresan, A Basic course in Real Analysis, CRC Press, 2014.
3. K. G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
4. G. B. Thomas, Calculus, 12th Edition 2009

USMT 202: DISCRETE MATHEMATICS

Unit I: Preliminary Counting (15 Lectures)

(1) Finite and infinite sets, countable and uncountable sets examples such as $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{N}\times\mathbb{N}$, $\mathbb{Q}$ $(0,1)$, $\mathbb{R}$.

(2) Addition and multiplication Principle, counting sets of pairs, two ways counting.

(3) Stirling numbers of second kind. Simple recursion formulae satisfied by $S(n,k)$ for $k = 1,2,\cdots,n-1,n$.

(4) Pigeonhole principle simple and strong form and examples, its applications to geometry.

Unit II: Advanced Counting (15 Lectures)

(1) Permutation and combination of sets and multi-sets, circular permutations, emphasis on solving problems.

(2) Binomial and Multinomial Theorem, Pascal identity, examples of standard identities such as the following with emphasis on combinatorial proofs.

- $\sum_{k=0}^{r}\binom{m}{k}\binom{n}{r-k}=\binom{m+n}{r}$
- $\sum_{i=0}^{k}\binom{k}{i}^2=\binom{2k}{k}$
- $\sum_{i=r}^{n}\binom{i}{r}=\binom{n+1}{r+1}$
- $\sum_{i=0}^{n}\binom{n}{i}=2^n$

(3) Non-negative integer solutions of equation $x_1+x_2+\cdots+x_k=n$.

(4) Principal of inclusion and exclusion, its applications, derangements, explicit formula for d_n, deriving formula for Euler's function $\phi(n)$.

Unit III: Permutations and Recurrence relation (15 lectures)

(1) Permutation of objects, S_n, composition of permutations, results such as every permutation is a product of disjoint cycles, every cycle is a product of transpositions, signature of a permutation, even and odd permutations, cardinality of S_n, A_n.

(2) Recurrence Relations, definition of homogeneous, non-homogeneous, linear, non-linear recurrence relation, obtaining recurrence relations of Tower of Hanoi, Fibonacci sequence, etc. in counting problems, solving homogeneous as well as non homogeneous recurrence relations by using iterative methods, solving a homogeneous recurrence relation of second degree using algebraic method proving the necessary result.

Recommended Books:

1. Norman Biggs, Discrete Mathematics, Oxford University Press.
2. Richard Brualdi, Introductory Combinatorics, John Wiley and sons.
3. V. Krishnamurthy, Combinatorics-Theory and Applications, Affiliated East West Press.
4. Discrete Mathematics and its Applications, Tata McGraw Hills.
5. Schaum's outline series, Discrete mathematics,
6. Allen Tucker, Applied Combinatorics, John Wiley and Sons.
7. Sharad Sane, Combinatorial Techniques, Springer.

PRACTICALS FOR F.Y.B.Sc
USMTP02-Practicals

A. **Practicals for USMT201 :**

(1) Limit of a function and Sandwich theorem, Continuous and discontinuous function.

(2) Algebra of limits and continuous functions, Intermediate Value theorem, Bolzano-Weierstrass theorem.

(3) Properties of differentiable functions, derivatives of inverse functions and implicit functions.

(4) Higher order derivatives, Leibnitz Rule.

(5) Mean value theorems and its applications, L'Hospital's Rule, Increasing and Decreasing functions.

(6) Extreme values, Taylor's Theorem and Curve Sketching.

(7) Miscellaneous Theoretical Questions based on full paper.

B. **Practicals for USMT202:**

(1) Counting principles, Two way counting.

(2) Stirling numbers of second kind, Pigeon hole principle.

(3) Multinomial theorem, identities, permutation and combination of multi-set.

(4) Inclusion-Exclusion principle. Euler phi function.

(5) Composition of permutations, signature of permutation, inverse of permutation.

(6) Recurrence relation.

(7) Miscellaneous Theoretical Questions based on full paper.

TUTORIALS FOR F.Y.B.A

Tutorials for UAMT201 :

(1) Limit of a function and Sandwich theorem, Continuous and discontinuous function.

(2) Algebra of limits and continuous functions, Intermediate Value theorem, Bolzano-Weierstrass theorem.

(3) Properties of differentiable functions, derivatives of inverse functions and implicit functions.

(4) Higher order derivatives, Leibnitz Rule.

(5) Mean value theorems and its applications, L'Hospital's Rule, Increasing and Decreasing functions.

(6) Extreme values, Taylor's Theorem and Curve Sketching.

(7) Miscellaneous Theoretical Questions based on full paper.

Scheme of Examination (75:25)

The performance of the learners shall be evaluated into two parts. The learner's performance shall be assessed by Internal Assessment with 25 percent marks in the first part and by conducting the Semester End Examinations with 75 percent marks in the second part. The allocation of marks for the Internal Assessment and Semester End Examinations are as shown below:-

I. **Internal Evaluation of 25 Marks:**

F.Y.B.Sc. :

(i) One class Test of 20 marks to be conducted during Practical session.
Paper pattern of the Test:
Q1: Definitions/ Fill in the blanks/ True or False with Justification (04 Marks).
Q2: Multiple choice 5 questions. (10 Marks: 5×2)
Q3: Attempt any 2 from 3 descriptive questions. (06 marks: 2×3)

(ii) Active participation in routine class: 05 Marks.

F.Y.B.A. :

(i) One class Test of 20 marks to be conducted during Tutorial session.
Paper pattern of the Test:
Q1: Definitions/ Fill in the blanks/ True or False with Justification (04 Marks).
Q2: Multiple choice 5 questions. (10 Marks: 5×2)
Q3: Attempt any 2 from 3 descriptive questions. (06 marks: 2×3)

(ii) Journal : 05 Marks.

II. **Semester End Theory Examinations :** There will be a Semester-end external Theory examination of 75 marks for each of the courses USMT101/UAMT101, USMT102 of Semester I and USMT201/UAMT201, USMT202 of semester II to be conducted by the college.

1. Duration: The examinations shall be of 2 and $\frac{1}{2}$ hours duration.
2. Theory Question Paper Pattern:
   a) There shall be FOUR questions. The first three questions Q1, Q2, Q3 shall be of 20 marks, each based on the units I, II, III respectively. The question Q4 shall be of 15 marks based on the entire syllabus.
   b) All the questions shall be compulsory. The questions Q1, Q2, Q3, Q4 shall have internal choices within the questions. Including the choices, the marks for each question shall be 25-27.
   c) The questions Q1, Q2, Q3, Q4 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.
3. **Semester End Examinations Practicals:**
   At the end of the Semesters I & II Practical examinations of three hours duration and 100 marks shall be conducted for the courses USMTP01, USMTP02.
   In semester I, the Practical examinations for USMT101 and USMT102 are held together by the college.
   In Semester II, the Practical examinations for USMT201 and USMT202 are held together by the college.

   Paper pattern: The question paper shall have two parts A and B.
   Each part shall have two Sections.

   Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions (04 objective questions from each unit) ($8 \times 3 = 24$ Marks).

   Section II Problems: Attempt any Two out of Three (01 descriptive question from each unit) ($8 \times 2 = 16$ Marks).

Practical Course	Part A	Part B	Marks out of	duration
USMTP01	Questions from USMT101	Questions from USMT102	80	3 hours
USMTP02	Questions from USMT201	Questions from USMT202	80	3 hours

Marks for Journals and Viva:
For each course USMTP01 (USMT101, USMT102) and USMTP02 (USMT201, USMT202):

1. Journal: 10 marks (5 marks for each journal).
2. Viva: 10 marks.

Each Practical of every course of Semester I and II shall contain at least 10 objective questions and at least 6 descriptive questions.
A student must have a certified journal before appearing for the practical examination.
In case a student does not posses a certified journal he/she will be evaluated for 80 marks.
He/she is not qualified for Journal + Viva marks.

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UNIVERSITY OF MUMBAI

Syllabus

for S. Y. B. Sc. / S. Y. B. A. Semester III & IV (CBCS)

Program: B. Sc. / B. A.

Course: Mathematics

with effect from the academic year 2021-2022

(UNIVERSITY OF MUMBAI)

Syllabus for: S.Y.B.Sc./S.Y.B.A.

Program: B.Sc./B/A.
Course: Mathematics
Choice based Credit System (CBCS)
with effect from the
academic year 2021-22

SEMESTER III

Calculus III				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 301, UAMT 301	I	Infinite Series	2	3
	II	Riemann Integration		
	III	Applications of Integrations and Improper Integrals		
Linear Algebra I				
USMT 302 ,UAMT 302	I	System of Equations and Matrices	2	3
	II	Vector Spaces over IR		
	III	Determinants, Linear Equations (Revisited)		
ORDINARY DIFFERENTIAL EQUATIONS				
USMT 303	I	Higher Order linear Differential Equations	2	3
	II	Systems of First Order Linear differential equations		
	III	Numerical Solutions of Ordinary Differential Equations		
PRACTICALS				
USMTP03		Practicals based on USMT301, USMT 302 and USMT 303	3	5
UAMTP03		Practicals based on UAMT301, UAMT 302	2	4

SEMESTER IV

Multivariable Calculus I				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 401, UAMT 401	I	Functions of several variables	2	3
	II	Differentiation of Scalar Fields		
	III	Applications of Differentiation of Scalar Fields and Differentiation of Vector Fields		
Linear Algebra II				
USMT 402 ,UAMT 402	I	Linear transformation, Isomorphism, Matrix associated with L.T.	2	3
	II	Inner product spaces		
	III	Eigen values, eigen vectors, diagonalizable matrix		
Numerical methods (Elective A)				
USMT 403A	I	Solutions of algebraic and transcendental equations	2	3
	II	Interpolation, Curve fitting, Numerical integration		
	III	Solutions of linear system of Equations and eigen value problems		
Statistical methods an their applications(Elective B)				
USMT 403B	I	Descriptive Statistics and random variables	2	3
	II	Probability Distribution and Correlation		
	III	Inferential Statistics		
PRACTICALS				
USMTP04		Practicals based on USMT401, USMT 402 and USMT 403	3	5
UAMTP04		Practicals based on UAMT401, UAMT 402	2	4

Teaching Pattern for Semester III

1. Three lectures per week per course. Each lecture is of 48 minutes duration.

2. One Practical (2L) per week per batch for courses USMT301, USMT 302 combined and one Practical (3L) per week for course USMT303 (the batches tobe formed as prescribed by the University. Each practical session is of 48 minutes duration.)

Teaching Pattern for Semester IV

1. Three lectures per week per course. Each lecture is of 48 minutes duration.

2. One Practical (2L) per week per batch for courses USMT301, USMT 302 combined and one Practical (3L) per week for course USMT303 (the batches to be formed as prescribed by the University. Each practical session is of 48 minutes duration.)

Semester-III

Note: Unless indicated otherwise, proofs of the results mentioned in the syllabus should be covered.

USMT301/ UAMT301: Calculus III

Unit I. Infinite Series (15 Lectures)

1. Infinite series in $\mathbb{R}$. Definition of convergence and divergence. Basic examples including geometric series. Elementary results such as if $\sum_{n=1}^{\infty} a_n$ is convergent, then $a_n \longrightarrow 0$ but converse not true. Cauchy Criterion. Algebra of convergent series.

2. Tests for convergence: Comparison Test, Limit Comparison Test, Ratio Test (without proof), Root Test (without proof), Abel Test (without proof) and Dirichlet Test (without proof). Examples. The decimal expansion of real numbers. Convergence of $\sum_{n=1}^{\infty} \frac{1}{n^p}$ $(p > 1)$. Divergence of harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$.

3. Alternating series. Leibnitz's Test. Examples. Absolute convergence, absolute convergence implies convergence but not conversely. Conditional Convergence.

Unit II. Riemann Integration (15 Lectures)

1. Idea of approximating the area under a curve by inscribed and circumscribed rectangles. Partitions of an interval. Refinement of a partition. Upper and Lower sums for a bounded real valued function on a closed and bounded interval. Riemann integrability and the Riemann integral.

2. Criterion for Riemann integrability. Characterization of the Riemann integral as the limit of a sum. Examples.

3. Algebra of Riemann integrable functions. Also, basic results such as if $f : [a, b] \longrightarrow \mathbb{R}$ is integrable, then (i) $\int_a^b f(x)\ dx = \int_a^c f(x)\ dx + \int_c^b f(x)\ dx$. (ii) $|f|$ is integrable and $\left| \int_a^b f(x)\ dx \right| \leq \int_a^b |f|(x)\ dx$ (iii) If $f(x) \geq 0$ for all $x \in [a, b]$ then $\int_a^b f(x)\ dx \geq 0$.
4. Riemann integrability of a continuous function, and more generally of a bounded function whose set of discontinuities has only finitely many points. Riemann integrability of monotone functions.

Unit III. Applications of Integrations and Improper Integrals (15 lectures)

1. Area between the two curves. Lengths of plane curves. Surface area of surfaces of revolution.
2. Continuity of the function $F(x) = \int_a^x f(t)\ dt, x \in [a, b]$, when $f : [a, b] \longrightarrow \mathbb{R}$ is Riemann integrable. First and Second Fundamental Theorems of Calculus.
3. Mean value theorem. Integration by parts formula. Leibnitz's Rule.
4. Definition of two types of improper integrals. Necessary and sufficient conditions for convergence.
5. Absolute convergence. Comparison and limit comparison tests for convergence.
6. Gamma and Beta functions and their properties. Relationship between them (without proof).

Reference Books

1. Sudhir Ghorpade, Balmohan Limaye; A Course in Calculus and Real Analysis (second edition); Springer.
2. R.R. Goldberg; Methods of Real Analysis; Oxford and IBH Pub. Co., New Delhi, 1970.
3. Calculus and Analytic Geometry (Ninth Edition); Thomas and Finney; Addison-Wesley, Reading Mass., 1998.
4. T. Apostol; Calculus Vol. 2; John Wiley.

Additional Reference Books

1. Ajit Kumar, S.Kumaresan; A Basic Course in Real Analysis; CRC Press, 2014
2. D. Somasundaram and B.Choudhary; A First Course in Mathematical Analysis, Narosa, New Delhi, 1996.
3. K. Stewart; Calculus, Booke/Cole Publishing Co, 1994.
4. J. E. Marsden, A.J. Tromba and A. Weinstein; Basic Multivariable Calculus; Springer.
5. R.G. Brtle and D. R. Sherbert; Introduction to Real Analysis Second Ed. ; John Wiley, New Yorm, 1992.

6. M. H. Protter; Basic Elements of Real Analysis; Springer-Verlag, New York, 1998.

USMT/UAMT 302: Linear Algebra I

Unit I. System of Equations, Matrices (15 Lectures)

1. Systems of homogeneous and non-homogeneous linear equations, Simple examples of finding solutions of such systems. Geometric and algebraic understanding of the solutions. Matrices (with real entries), Matrix representation of system of homogeneous and non-homogeneous linear equations. Algebra of solutions of systems of homogeneous linear equations. A system of homogeneous linear equations with number of unknowns more than the number of equations has infinitely many solutions.

2. Elementary row and column operations. Row equivalent matrices. Row reduction (of a matrix to its row echelon form). Gaussian elimination. Applications to solving systems of linear equations. Examples.

3. Elementary matrices. Relation of elementary row operations with elementary matrices. Invertibility of elementary matrices. Consequences such as (i) a square matrix is invertible if and only if its row echelon form is invertible. (ii) invertible matrices are products of elementary matrices. Examples of the computation of the inverse of a matrix using Gauss elimination method.

Unit II. Vector space over $\mathbb{R}$ (15 Lectures)

1. Definition of a vector space over $\mathbb{R}$. Subspaces; criterion for a nonempty subset to be a subspace of a vector space. Examples of vector spaces, including the Euclidean space $\mathbb{R}^n$, lines, planes and hyperplanes in $\mathbb{R}^n$ passing through the origin, space of systems of homogeneous linear equations, space of polynomials, space of various types of matrices, space of real valued functions on a set.

2. Intersections and sums of subspaces. Direct sums of vector spaces. Quotient space of a vector space by its subspace.

3. Linear combination of vectors. Linear span of a subset of a vector space. Definition of a finitely generated vector space. Linear dependence and independence of subsets of a vector space.

4. Basis of a vector space. Basic results that any two bases of a finitely generated vector space have the same number of elements. Dimension of a vector space. Examples. Bases of a vector space as a maximal linearly independent sets and as minimal generating sets.

Unit III. Determinants, Linear Equations (Revisited) (15 Lectures)

1. Inductive definition of the determinant of a $n \times n$ matrix (e. g. in terms of expansion along the first row). Example of a lower triangular matrix. Laplace expansions along an arbitrary row or column. Determinant expansions using permutations
$\left(\det(A) = \sum_{\sigma \in S_n} \operatorname{sign}(\sigma) \prod_{i=1}^{n} a_{\sigma(i),i} \right)$.

2. Basic properties of determinants (Statements only); (i) $\det A = \det A^T$. (ii) Multilinearity and alternating property for columns and rows. (iii) A square matrix A is invertible if and only if $\det A \neq 0$. (iv) Minors and cofactors. Formula for A^{-1} when $\det A \neq 0$. (v) $\det(AB) = \det A \det B$.

3. Row space and the column space of a matrix as examples of vector space. Notion of row rank and the column rank. Equivalence of the row rank and the column rank. Invariance of rank upon elementary row or column operations. Examples of computing the rank using row reduction.

4. Relation between the solutions of a system of non-homogeneous linear equations and the associated system of homogeneous linear equations. Necessary and sufficient condition for a system of non-homogeneous linear equations to have a solution [viz., the rank of the coefficient matrix equals the rank of the augmented matrix $[A|B]$]. Equivalence of statements (in which A denotes an $n \times n$ matrix) such as the following.

   (i) The system $A\boldsymbol{x} = \boldsymbol{b}$ of non-homogeneous linear equations has a unique solution.

   (ii) The system $A\boldsymbol{x} = \mathbf{0}$ of homogeneous linear equations has no nontrivial solution.

   (iii) A is invertible.

   (iv) $\det A \neq 0$.

   (v) $\mathrm{rank}(A) = n$.

5. Cramers Rule. LU Decomposition. If a square matrix A is a matrix that can be reduced to row echelon form U by Gauss elimination without row interchanges, then A can be factored as $A = LU$ where L is a lower triangular matrix.

Reference books

1 Howard Anton, Chris Rorres, Elementary Linear Algebra, Wiley Student Edition).

2 Serge Lang, Introduction to Linear Algebra, Springer.

3 S Kumaresan, Linear Algebra - A Geometric Approach, PHI Learning.

4 Sheldon Axler, Linear Algebra done right, Springer.

5 Gareth Williams, Linear Algebra with Applications, Jones and Bartlett Publishers.

6 David W. Lewis, Matrix theory.

USMT303: Ordinary Differential Equations

Unit I. Higher order Linear Differential equations (15 Lectures)

1. The general $n-$th order linear differential equations, Linear independence, An existence and uniqueness theorem, the Wronskian, Classification: homogeneous and non-homogeneous, General solution of homogeneous and non-homogeneous LDE, The Differential operator and its properties.

2. Higher order homogeneous linear differential equations with constant coefficients, the auxiliary equations, Roots of the auxiliary equations: real and distinct, real and repeated, complex and complex repeated.

3. Higher order homogeneous linear differential equations with constant coefficients, the method of undermined coefficients, method of variation of parameters.

4. The inverse differential operator and particular integral, Evaluation of $\frac{1}{f(D)}$ for the functions like e^{ax}, $\sin ax$, $\cos ax$, x^m, $x^m \sin ax$, $x^m \cos ax$, $e^{ax}V$ and xV where V is any function of x,

5. Higher order linear differential equations with variable coefficients:
The Cauchy's equation: $x^3\frac{d^3y}{dx^3} + x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = f(x)$ and
The Legendre's equation: $(ax+b)^3\frac{d^3y}{dx^3} + (ax+b)^2\frac{d^2y}{dx^2} + (ax+b)\frac{dy}{dx} + y = f(x)$.

Reference Books

1. Units 5, 6, 7 and 8 of E.D. Rainville and P.E. Bedient; Elementary Differential Equations; Macmillan.

2. Units 5, 6 and 7 of M.D. Raisinghania; Ordinary and Partial Differential Equations; S. Chand.

Unit II. Systems of First Order Linear Differential Equations (15 Lectures)

(a) Existence and uniqueness theorem for the solutions of initial value problems for a system of two first order linear differential equations in two unknown functions x, y of a single independent variable t, of the form $\begin{cases} \frac{dx}{dt} = F(t,x,y) \\ \frac{dy}{dt} = G(t,x,y) \end{cases}$ (Statement only).

(b) Homogeneous linear system of two first order differential equations in two unknown functions of a single independent variable t, of the form $\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y, \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y. \end{cases}$.

(c) Wronskian for a homogeneous linear system of first order linear differential equations in two functions x, y of a single independent variable t. Vanishing properties of the Wronskian. Relation with linear independence of solutions.

(d) Homogeneous linear systems with constant coefficients in two unknown functions x, y of a single independent variable t. Auxiliary equation associated to a homogenous system of equations with constant coefficients. Description fo the general solution depending on the roots and their multiplicities of the auxiliary equation, proof of independence of the solutions. Real form of solutions in case the auxiliary equation has complex roots.

(e) Non-homogeneous linear system of linear system of two first order differential equations in two unknown functions of a single independent variable t, of the form
$$\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y + f_1(t), \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y + f_2(t). \end{cases}$$
General Solution of non-homogeneous system. Relation between the solutions of a system

of non-homogeneous linear differential equations and the associated system of homogeneous linear differential equations.

Reference Books

1. G.F. Simmons; Differential Equations with Applications and Historical Notes; Taylor's and Francis.

Unit III. Numerical Solution of Ordinary Differential Equations (15 lectures)

1. Numerical Solution of initial value problem of first order ordinary differential equation using:
   (i) Taylor's series method,
   (ii) Picard's method for successive approximation and its convergence,
   (iii) Euler's method and error estimates for Euler's method,
   (iv) Modified Euler's Method,
   (v) Runge-Kutta method of second order and its error estimates,
   (vi) Runge-Kutta fourth order method.
2. Numerical solution of simultaneous and higher order ordinary differential equation using:
   (i) Runge-Kutta fourth order method for solving simultaneous ordinary differential equation,
   (ii) Finite difference method for the solution of two point linear boundary value problem.

Reference Books

1. Units 8 of S. S. Sastry, Introductory Methods of Numerical Analysis, PHI.

Additional Reference Books

1. E.D. Rainville and P.E. Bedient, Elementary Differential Equations, Macmillan.
2. M.D. Raisinghania, Ordinary and Partial Differential Equations, S. Chand.
3. G.F. Simmons, Differential Equations with Applications and Historical Notes, Taylor's and Francis.
4. S. S. Sastry, Introductory Methods of Numerical Analysis, PHI.
5. K. Atkinson, W.Han and D Stewart, Numerical Solution of Ordinary Differential Equations, Wiley.

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USMT P03 / UAMT P03: Practicals

Suggested Practicals for USMT 301/ UAMT 301

1. Examples of convergent / divergent series and algebra of convergent series.
2. Tests for convergence of series.
3. Calculation of upper sum, lower sum and Riemann integral.
4. Problems on properties of Riemann integral.
5. Problems on fundamental theorem of calculus, mean value theorems, integration by parts, Leibnitz rule.
6. Convergence of improper integrals, different tests for convergence. Beta Gamma Functions.
7. Miscellaneous Theoretical Questions based on full paper.

Suggested Practicals for USMT302 / UAMT 302

1. Systems of homogeneous and non-homogeneous linear equations.
2. Elementary row/column operations and Elementary matrices.
3. Vector spaces, Subspaces.
4. Linear Dependence/independence, Basis, Dimension.
5. Determinant and Rank of a matrix.
6. Solution to a system of linear equations, LU decomposition
7. Miscellaneous Theory Questions.
8. Miscellaneous theory questions from units I, II and III.

Suggested Practicals For USMT 303

1. Finding the general solution of homogeneous and non-homogeneous higher order linear differential equations.
2. Solving higher order linear differential equations using method of undetermined coefficients and method of variation of parameters.
3. Solving a system of first order linear ODES have auxiliary equations with real and complex roots.
4. Finding the numerical solution of initial value problems using Taylor's series method, Picard's method, modified Euler's method, Runge-Kutta method of fourth order and calculating their accuracy.
5. Finding the numerical solution of simultaneous ordinary differential equation using fourth order Runge-Kutta method.
6. Finding the numerical solution of two point linear boundary value problem using Finite difference method.

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Semester-IV

Note: Unless indicated otherwise, proofs of the results mentioned in the syllabus should be covered.

USMT 401/ UAMT 401: Multivariable Calculus I

UNIT I. Functions of Several Variables (15 Lectures)

1. Review of vectors in $\mathbb{R}^n$ [with emphasis on $\mathbb{R}^2$ and $\mathbb{R}^3$] and basic notions such as addition and scalar multiplication, inner product, length (norm), and distance between two points.
2. Real-valued functions of several variables (Scalar fields). Graph of a function. Level sets (level curves, level surfaces, etc). Examples. Vector valued functions of several variables (Vector fields). Component functions. Examples.
3. Sequences, Limits and Continuity: Sequence in $\mathbb{R}^n$ [with emphasis on $\mathbb{R}^2$ and $\mathbb{R}^3$] and their limits. Neighbourhoods in $\mathbb{R}^n$. Limits and continuity of scalar fields. Composition of continuous functions. Sequential characterizations. Algebra of limits and continuity (Results with proofs). Iterated limits.
   Limits and continuity of vector fields. Algebra of limits and continuity vector fields. (without proofs).
4. Partial and Directional Derivatives of scalar fields: Definitions of partial derivative and directional derivative of scalar fields (with emphasis on $\mathbb{R}^2$ and $\mathbb{R}^3$). Mean Value Theorem of scalar fields.

UNIT II. Differentiation of Scalar Fields (15 Lectures)

1. Differentiability of scalar fields (in terms of linear transformation). The concept of (total) derivative. Uniqueness of total derivative of a differentiable function at a point. Examples of functions of two or three variables. Increment Theorem. Basic properties including (i) continuity at a point of differentiability, (ii)existence of partial derivatives at a point of differentiability, and (iii) differentiability when the partial derivatives exist and are continuous.
2. Gradient. Relation between total derivative and gradient of a function. Chain rule. Geometric properties of gradient. Tangent planes.
3. Euler's Theorem.
4. Higher order partial derivatives. Mixed Partial Theorem (n=2).

UNIT III. Applications of Differentiation of Scalar Fields and Differentiation of Vector Fields (15 lectures)

1. Applications of Differentiation of Scalar Fields: The maximum and minimum rate of change of scalar fields. Taylor's Theorem for twice continuously differentiable functions. Notions of local maxima, local minima and saddle points. First Derivative Test. Examples. Hessian matrix. Second Derivative Test for functions of two variables. Examples. Method of Lagrange Multipliers.

2. Differentiation of Vector Fields: Differentiability and the notion of (total) derivative. Differentiability of a vector field implies continuity, Jacobian matrix. Relationship between total derivative and Jacobian matrix. The chain rule for derivative of vector fields (statements only).

Reference books

1. T. Apostol; Calculus, Vol. 2 (Second Edition); John Wiley.
2. Sudhir Ghorpade, Balmohan Limaye; A Course in Multivariable Calculus and Analysis (Second Edition); Springer.
3. Walter Rudin; Principles of Mathematical Analysis; McGraw-Hill, Inc.
4. J. E. Marsden, A.J. Tromba and A. Weinstein, Basic Multivariable Calculus; Springer.
5. D.Somasundaram and B.Choudhary; A First Course in Mathematical Analysis, Narosa, New Delhi, 1996.
6. K. Stewart; Calculus; Booke/Cole Publishing Co, 1994.

Additional Reference Books

1. Calculus and Analytic Geometry, G.B. Thomas and R. L. Finney, (Ninth Edition); Addison-Wesley, 1998.
2. Howard Anton; Calculus- A new Horizon,(Sixth Edition); John Wiley and Sons Inc, 1999.
3. S L Gupta and Nisha Rani; Principles of Real Analysis; Vikas Publishing house PVT LTD.
4. Shabanov, Sergei; Concepts in Calculus, III: Multivariable Calculus; University Press of Florida, 2012.
5. S C Malik and Savita Arora; Mathematical Analysis; New Age International Publishers.

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USMT402/UAMT402: Linear Algebra II

UNIT I. Linear Transformations

1. Definition of a linear transformation of vector spaces; elementary properties. Examples. Sums and scalar multiples of linear transformations. Composites of linear transformations. A Linear transformation of $V \longrightarrow W$, where V, W are vector spaces over $\mathbb{R}$ and V is a finite-dimensional vector space is completely determined by its action on an ordered basis of V.

2. Null-space (kernel) and the image (range) of a linear transformation. Nullity and rank of a linear transformation. Rank-Nullity Theorem (Fundamental Theorem of Homomorphisms).

3. Matrix associated with linear transformation of $V \longrightarrow W$ where V and W are finite dimensional vector spaces over $\mathbb{R}$.. Matrix of the composite of two linear transformations. Invertible linear transformations (isomorphisms), Linear operator, Effect of change of bases on matrices of linear operator.

4. Equivalence of the rank of a matrix and the rank of the associated linear transformation. Similar matrices.

UNIT II. Inner Products and Orthogonality

1. Inner product spaces (over $\mathbb{R}$). Examples, including the Euclidean space $\mathbb{R}^n$ and the space of real valued continuous functions on a closed and bounded interval. Norm associated to an inner product. Cauchy-Schwarz inequality. Triangle inequality.

2. Angle between two vectors. Orthogonality of vectors. Pythagoras theorem and some geometric applications in $\mathbb{R}^2$. Orthogonal sets, Orthonormal sets. Gram-Schmidt orthogonalizaton process. Orthogonal basis and orthonormal basis for a finite-dimensional inner product space.

3. Orthogonal complement of any set of vectors in an inner product space. Orthogonal complement of a set is a vector subspace of the inner product space. Orthogonal decomposition of an inner product space with respect to its subspace. Orthogonal projection of a vector onto a line (one dimensional subspace). Orthogonal projection of an inner product space onto its subspace.

UNIT III. Eigenvalues, Eigenvectors and Diagonalisation

1. Eigenvalues and eigenvectors of a linear transformation of a vector space into itself and of square matrices. The eigenvectors corresponding to distinct eigenvalues of a linear transformation are linearly independent. Eigen spaces. Algebraic and geometric multiplicity of an eigenvalue.

2. Characteristic polynomial. Properties of characteristic polynomials (only statements). Examples. Cayley-Hamilton Theorem. Applications.

3. Invariance of the characteristic polynomial and eigenvalues of similar matrices.

4. Diagonalisable matrix. A real square matrix A is diagonalisable if and only if there is a basis of $\mathbb{R}^n$ consisting of eigenvectors of A. (Statement only - $A_{n\times n}$ is diagonalisable if and only if sum of algebraic multiplicities is equal to sum of geometric multiplicities of all the eigenvalues of $A \; = n$). Procedure for diagonalising a matrix.

5. Spectral Theorem for Real Symmetric Matrices (Statement only). Examples of orthogonal diagonalisation of real symmetric matrices. Applications to quadratic forms and classification of conic sections.

Reference books

1. Howard Anton, Chris Rorres; Elementary Linear Algebra; Wiley Student Edition).

2. Serge Lang; Introduction to Linear Algebra; Springer.

3. S Kumaresan; Linear Algebra - A Geometric Approach; PHI Learning.

4. Sheldon Axler; Linear Algebra done right; Springer.

5. Gareth Williams; Linear Algebra with Applications; Jones and Bartlett Publishers.

6. David W. Lewis; Matrix theory.

USMT403A: Numerical Methods (Elective A)

Unit I. Solution of Algebraic and Transcendental Equations (15L)

1. Measures of Errors: Relative, absolute and percentage errors, Accuracy and precision: Accuracy to n decimal places, accuracy to n significant digits or significant figures, Rounding and Chopping of a number, Types of Errors: Inherent error, Round-off error and Truncation error.

2. Iteration methods based on first degree equation: Newton-Raphson method. Secant method. Regula-Falsi method.
   Derivations and geometrical interpretation and rate of convergence of all above methods to be covered.

3. General Iteration method: Fixed point iteration method.

Unit II. Interpolation, Curve fitting, Numerical Integration(15L)

1. Interpolation: Lagrange's Interpolation. Finite difference operators: Forward Difference operator, Backward Difference operator. Shift operator. Newton's forward difference interpolation formula. Newton's backward difference interpolation formula.
   Derivations of all above methods to be covered.

2. Curve fitting: linear curve fitting. Quadratic curve fitting.

3. Numerical Integration: Trapezoidal Rule. Simpson's 1/3 rd Rule. Simpson's 3/8th Rule. Derivations all the above three rules to be covered.

Unit III. Solution Linear Systems of Equations, Eigenvalue problems(15L)

1. Linear Systems of Equations: LU Decomposition Method (Dolittle's Method and Crout's Method). Gauss-Seidel Iterative method.

2. Eigenvalue problems: Jacobi's method for symmetric matrices. Rutishauser method for arbitrary matrices.

Reference Books:

1. Kendall E. and Atkinson; An Introduction to Numerical Analysis; Wiley.

2. M. K. Jain, S. R. K. Iyengar and R. K. Jain; Numerical Methods for Scientific and Engineering Computation; New Age International Publications.

3. S. Sastry; Introductory methods of Numerical Analysis; PHI Learning.

4. An introduction to Scilab-Cse iitb.

Additional Reference Books

1. S.D. Comte and Carl de Boor; Elementary Numerical Analysis, An algorithmic approach; McGrawHillll International Book Company.

2. Hildebrand F.B.; Introduction to Numerical Analysis; Dover Publication, NY.

3. Scarborough James B.; Numerical Mathematical Analysis; Oxford University Press, New Delhi.

USMT403B Statistical Methods and their Applications (Elective B)

Unit I. Descriptive Statistics and random variables (15 Lectures)
Measures of location (mean, median, mode), Partition values and their graphical locations, measures of dispersion, skewness and kurtosis, Exploratory Data Analysis (Five number summary, Box Plot, Outliers), Random Variables (discrete and continuous), Expectation and variance of a random variable.

Unit II. Probability Distributions and Correlation (15 Lectures)
Discrete Probability Distribution (Binomial, Poisson), Continuous Probability Distribution: (Uniform, Normal), Correlation, Karl Pearson's Coefficient of Correlation, Concept of linear Regression, Fitting of a straight line and curve to the given data by the method of least squares, relation between correlation coefficient and regression coefficients.

Unit III. Inferential Statistics (15 lectures)
Population and sample, parameter and statistic, sampling distribution of Sample mean and Sample Variance, concept of statistical hypothesis, critical region, level of significance, confidence interval and two types of errors, Tests of significance (t-test, Z-test, F-test, Chi-Square Test (only applications))

Reference Books

1. Fundamentals of Mathematical Statistics,12th Edition, S. C. Gupta and V. K. Kapoor,Sultan Chand & Sons, 2020.

2. Statistics for Business and Economics, 11th Edition, David R. Anderson, Dennis J. Sweeney and Thomas A. Williams, Cengage Learning, 2011.

3. Introductory Statistics, 8th Edition, Prem S. Mann, John Wiley & Sons Inc., 2013.

4. A First Course in Statistics, 12th Edition, James McClave and Terry Sincich, Pearson Education Limited, 2018.

5. Introductory Statistics, Barbara Illowsky, Susan Dean and Laurel Chiappetta, OpenStax, 2013.

6. Hands-On Programming with R, Garrett Grolemund, O'Reilly.

USMT P04 / UAMT P04: Practicals

Suggested Practical for USMT 401/ UAMT 401

1. Limits and continuity of scalar fields and vector fields, using "definition and otherwise" , iterated limits.
2. Computing directional derivatives, partial derivatives and mean value theorem of scalar fields.
3. Differentiability of scalar field,Total derivative, gradient, level sets and tangent planes.
4. Chain rule, higher order derivatives and mixed partial derivatives of scalar fields.
5. Maximum and minimum rate of change of scalar fields. Taylor's Theorem. Finding Hessian/Jacobean matrix. Differentiation of a vector field at a point. Chain Rule for vector fields.
6. Finding maxima, minima and saddle points. Second derivative test for extrema of functions of two variables and method of Lagrange multipliers.
7. Miscellaneous Theoretical Questions based on full paper.

Suggested Practicals for USMT402/UAMT 402

1. Linear transformation, Kernel, Rank-Nullity Theorem.
2. Linear Isomorphism, Matrix associated with Linear transformations.
3. Inner product and properties, Projection, Orthogonal complements.
4. Orthogonal, orthonormal sets, Gram-Schmidt orthogonalisation
5. Eigenvalues, Eigenvectors, Characteristic polynomial. Applications of Cayley Hamilton Theorem.
6. Diagonalisation of matrix, orthogonal diagonalisation of symmetric matrix and application to quadratic form.
7. Miscellaneous Theoretical Questions based on full paper.

Suggested Practicals for USMT403A

The Practical no. 1 to 6 should be performed either using non-programable scientific calculators or by using the software Scilab.

1. Newton-Raphson method, Secant method.
2. Regula-Falsi method, Iteration Method..
3. Interpolating polynomial by Lagrange's Interpolation, Newton forward and backward difference Interpolation.
4. Curve fitting, Trapezoidal Rule, Simpson's 1/3rd Rule, Simpson's 3/8th Rule.
5. LU decomposition method, Gauss-Seidel Interative method.

6. Jacobi's method, Rutishauser method..
7. Miscellaneous theoretical questions from all units.

Suggested Practicals for USMT403B
All practicals should be performed using any one of the following softwares: MS Excel, R, Strata, SPSS, Sage Math to carry out data analysis and computations.

1. Descriptive Statistics.
2. Random Variables.
3. Probability Distributions.
4. Correlation and Regression.
5. Testing of hypothesis.
6. Case studies.
7. Miscellaneous Theory questions based on Unit I,II,III.

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Scheme of Examination (75:25)

The performance of the learners shall be evaluated into two parts.

- Internal Assessment of 25 percent marks.
- Semester End Examinations of 75 percent marks.

I. **Internal Evaluation of 25 Marks:**

S.Y.B.Sc. :

(i) One class Test of 20 marks to be conducted during Practical session.
Paper pattern of the Test:
Q1: Definitions/ Fill in the blanks/ True or False with Justification (04 Marks).
Q2: Multiple choice 5 questions. (10 Marks: 5×2)
Q3: Attempt any 2 from 3 descriptive questions. (06 marks: 2×3)

(ii) Active participation in routine class: 05 Marks.

OR

Students who are willing to explore topics related to syllabus, dealing with applications historical development or some interesting theorems and their applications can be encouraged to submit a project for 25 marks under the guidance of teachers.

S.Y.B.A. :

(i) One class Test of 20 marks to be conducted during Tutorial session.
Paper pattern of the Test:
Q1: Definitions/ Fill in the blanks/ True or False with Justification (04 Marks).

Q2: Multiple choice 5 questions. (10 Marks: 5×2)

Q3: Attempt any 2 from 3 descriptive questions. (06 marks: 2×3)

(ii) Journal : 05 Marks.

OR

Students who are willing to explore topics related to syllabus, dealing with applications historical development or some interesting theorems and their applications can be encouraged to submit a project for 25 marks under the guidance of teachers.

II. **Semester End Theory Examinations :** There will be a Semester-end external Theory examination of 75 marks for each of the courses USMT301/UAMT301, USMT/USAT 302, USMT 303 of Semester III and USMT/UAMT401, USMT/UAMT 402, USMT 403 of semester IV to be conducted by the college.

1. Duration: The examinations shall be of 2 and $\frac{1}{2}$ hours duration.
2. Theory Question Paper Pattern:
    a) There shall be FOUR questions. The first three questions Q1, Q2, Q3 shall be of 20 marks, each based on the units I, II, III respectively. The question Q4 shall be of 15 marks based on the entire syllabus.
    b) All the questions shall be compulsory. The questions Q1, Q2, Q3, Q4 shall have internal choices within the questions. Including the choices, the marks for each question shall be 25-27.
    c) The questions Q1, Q2, Q3, Q4 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.

III. **Semester End Examinations Practicals:**

At the end of the Semesters III & IV Practical examinations of three hours duration and 150 marks shall be conducted for the courses USMTP03, USMTP04.

At the end of the Semesters III & IV Practical examinations of two hours duration and 100 marks shall be conducted for the courses UAMTP03, UAMTP04.
In semester III, the Practical examinations for USMT301/UAMT301, USMT302/UAMT302 and USMT303 are held together by the college.
In Semester IV, the Practical examinations for USMT401/UAMT401, USMT402/UAMT402 and USMT403 are held together by the college.

Paper pattern: The question paper shall have two parts A and B.
Each part shall have two Sections.

Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions (04 objective questions from each unit) ($8 \times 3 = 24$ Marks).

Section II Problems: Attempt any Two out of Three (01 descriptive question from each unit) ($8 \times 2 = 16$ Marks).

Practical Course	Part A	Part B	Part C	Marks out of	duration
USMTP03	Questions from USMT301	Questions from USMT302	Questions from USMT 303	120	3 hours
UAMTP03	Questions from UAMT301	Questions from UAMT302	——	80	2 hours
USMTP04	Questions from USMT401	Questions from USMT402	Questions from USMT403	120	3 hours
UAMTP04	Questions from UAMT401	Questions from UAMT402	——	80	2 hours

Marks for Journals and Viva:
For each course USMT301/UAMT301, USMT302/UAMT302, USMT303, USMT401/UAMT401, USMT402/UAMT402, USMT3031:

1. Journal: 10 marks (5 marks for each journal).
2. Viva: 10 marks.

Each Practical of every course of Semester III and IV shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the journal. .
A student must have a certified journal before appearing for the practical examination.
In case a student does not posses a certified journal he/she will be evaluated for 120/80 marks. He/she is not qualified for Journal + Viva marks.

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UNIVERSITY OF MUMBAI
No. UG/72 of 2018-19

CIRCULAR:-

Attention of the Principals of the affiliated Colleges and Directors of the recognized Institutions in Humanities and Sci. & Tech. Faculty is invited to this office Circular No. UG/488 of 2009 dated 22nd December, 2009, relating to syllabus of the B.A./B.Sc. degree course and Circular No. UG/90 of 2012-13, dated 8th November, 2012, relating to syllabus of the B.Sc. degree course.

They are hereby informed that the recommendations made by the Board of Studies in Mathematics at its meeting held on 3rd May, 2018 have been accepted by the Academic Council at its meeting held on 5th May, 2018 **vide** item No. 4.74 and that in accordance therewith, the revised syllabus for the S.Y.B.Sc./S.Y.B.A. in Mathematics (Sem - III & IV), has been brought into force with effect from the academic year 2018-19, accordingly. (The same is available on the University's website www.mu.ac.in).

MUMBAI – 400 032
6th ~~June~~ July, 2018

(Dr. Dinesh Kamble)
I/c REGISTRAR

To

The Principals of the affiliated Colleges & Directors of the recognized Institutions in Humanities and Sci. & Tech. Faculty. (Circular No. UG/334 of 2017-18 dated 9th January, 2018.)

A.C/4.74/05/05/2018

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No. UG/72 -A of 2018 MUMBAI-400 032 6th ~~June~~ July, 2018

Copy forwarded with Compliments for information to:-

1) The I/c Dean, Faculty of Humanities and Science & Technology,
2) The Chairman, Board of Studies in Mathematics,
3) The Director, Board of Examinations and Evaluation,
4) The Director, Board of Students Development,
5) The Co-Ordinator, University Computerization Centre,

(Dr. Dinesh Kamble)
I/c REGISTRAR

(UNIVERSITY OF MUMBAI)

Syllabus for: S.Y.B.Sc./S.Y.B.A.

Program: B.Sc./B/A.
Course: Mathematics
Choice based Credit System (CBCS)
with effect from the
academic year 2018-19

SEMESTER III

CALCULUS III				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 301, UAMT 301	I	Functions of several variables	2	3
	II	Differentiation		
	III	Applications		
ALGEBRA III				
USMT 302 ,UAMT 302	I	Linear Transformations and Matrices	2	3
	II	Determinants		
	III	Inner Product Spaces		
DISCRETE MATHEMATICS				
USMT 303	I	Permutations and Recurrence Relation	2	3
	II	Preliminary Counting		
	III	Advanced Counting		
PRACTICALS				
USMTP03		Practicals based on USMT301, USMT 302 and USMT 303	3	5
UAMTP03		Practicals based on UAMT301, UAMT 302	2	4

SEMESTER IV

CALCULUS IV				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 401, UAMT 401	I	Riemann Integration	2	3
	II	Indefinite Integrals and Improper Integrals		
	III	Beta and Gamma Functions And Applications		
ALGEBRA IV				
USMT 402 ,UAMT 402	I	Groups and Subgroups	2	3
	II	Cyclic Groups and Cyclic subgroups		
	III	Lagrange's Theorem and Group Homomorphism		
ORDINARY DIFFERENTIAL EQUATIONS				
USMT 403	I	First order First degree Differential equations	2	3
	II	Second order Linear Differential equations		
	III	Linear System of Ordinary Differential Equations		
PRACTICALS				
USMTP04		Practicals based on USMT401, USMT 402 and USMT 403	3	5
UAMTP04		Practicals based on UAMT401, UAMT 402	2	4

Teaching Pattern for Semester III

1. Three lectures per week per course. Each lecture is of 48 minutes duration.
2. One Practical (2L) per week per batch for courses USMT301, USMT 302 combined and one Practical (3L) per week for course USMT303 (the batches tobe formed as prescribed by the University. Each practical session is of 48 minutes duration.)

Teaching Pattern for Semester IV

1. Three lectures per week per course. Each lecture is of 48 minutes duration.
2. One Practical (2L) per week per batch for courses USMT301, USMT 302 combined and one Practical (3L) per week for course USMT303 (the batches tobe formed as prescribed by the University. Each practical session is of 48 minutes duration.)

S.Y.B.Sc. / S.Y.B.A. Mathematics

SEMESTER III

USMT 301, UAMT 301: CALCULUS III

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

Unit I: Functions of several variables (15 Lectures)

1. The Euclidean inner product on $\mathbb{R}^n$ and Euclidean norm function on $\mathbb{R}^n$, distance between two points, open ball in $\mathbb{R}^n$, definition of an open subset of $\mathbb{R}^n$, neighbourhood of a point in $\mathbb{R}^n$, sequences in $\mathbb{R}^n$, convergence of sequences- these concepts should be specifically discussed for $n = 3$ and $n = 3$.
2. Functions from $\mathbb{R}^n \longrightarrow \mathbb{R}$ (scalar fields) and from $\mathbb{R}^n \longrightarrow \mathbb{R}^m$ (vector fields), limits, continuity of functions, basic results on limits and continuity of sum, difference, scalar multiples of vector fields, continuity and components of a vector fields.
3. Directional derivatives and partial derivatives of scalar fields.
4. Mean value theorem for derivatives of scalar fields.

Reference for Unit I:
Sections 8.1, 8.2, 8.3, 8.4, 8.5, 8.6, 8.7, 8.8, 8.9, 8.10 of Calculus, Vol. 2 (Second Edition) by Apostol.

Unit II: Differentiation (15 Lectures)

1. Differentiability of a scalar field at a point of $\mathbb{R}^n$ (in terms of linear transformation) and on an open subset of $\mathbb{R}^n$, the total derivative, uniqueness of total derivative of a differentiable function at a point, simple examples of finding total derivative of functions such as $f(x, y) = x^2 + y^2, f(x, y, z) = x + y + z,$, differentiability at a point of a function f implies continuity and existence of direction derivatives of f at the point, the existence of continuous partial derivatives in a neighbourhood of a point implies differentiability at the point.

2. Gradient of a scalar field, geometric properties of gradient, level sets and tangent planes.
3. Chain rule for scalar fields.
4. Higher order partial derivatives, mixed partial derivatives, sufficient condition for equality of mixed partial derivative.

Reference for Unit II:
Sections 8.11, 8.12, 8.13, 8.14, 8.15, 8.16, 8.17, 8.23 of Calculus, Vol.2 (Second Edition) by T. Apostol, John Wiley.

Unit III: Applications (15 lectures)

1. Second order Taylor's formula for scalar fields.
2. Differentiability of vector fields, definition of differentiability of a vector field at a point, Jacobian matrix, differentiability of a vector field at a point implies continuity. The chain rule for derivative of vector fields (statements only)
3. Mean value inequality.
4. Hessian matrix, Maxima, minima and saddle points.
5. Second derivative test for extrema of functions of two variables.
6. Method of Lagrange Multipliers.

Reference for Unit III:
Sections 8.18, 8.19, 8.20, 8.21, 8.22, 9.9, 9.10, 9.11, 9.12, 9.13, 9.14 9.13, 9.14 from Apostol, Calculus Vol. 2, (Second Edition) by T. Apostol.

Recommended Text Books:

1. T. Apostol: Calculus, Vol. 2, John Wiley.
2. J. Stewart, Calculus, Brooke/ Cole Publishing Co.

Additional Reference Books

(1) G.B. Thoman and R. L. Finney, Calculus and Analytic Geometry, Ninth Edition, Addison-Wesley, 1998.

(2) Sudhir R. Ghorpade and Balmohan V. Limaye, A Course in Multivariable Calculus and Analysis, Springer International Edition.

(3) Howard Anton, Calculus- A new Horizon, Sixth Edition, John Wiley and Sons Inc, 1999.

USMT 302/UAMT 302: ALGEBRA III

Note: Revision of relevant concepts is necessary.

Unit 1: Linear Transformations and Matrices (15 lectures)

1. Review of linear transformations: Kernel and image of a linear transformation, Rank-Nullity theorem (with proof), Linear isomorphisms, inverse of a linear isomorphism, Any $n-$ dimensional real vector space is isomorphic to $\mathbb{R}^n$.

2. The matrix units, row operations, elementary matrices, elementary matrices are invertible and an invertible matrix is a product of elementary matrices.

3. Row space, column space of an $m \times n$ matrix, row rank and column rank of a matrix, Equivalence of the row and the column rank, Invariance of rank upon elementary row or column operations.

4. Equivalence of rank of an $m \times n$ matrix A and rank of the linear transformation $L_A : \mathbb{R}^n \longrightarrow \mathbb{R}^m \quad (L_A(X) = AX)$. The dimension of solution space of the system of linear equations $AX = 0$ equals $n-\text{rank}(A)$.

5. The solutions of non-homogeneous systems of linear equations represented by $AX = B$, Existence of a solution when rank$(A)=$ rank(A, B), The general solution of the system is the sum of a particular solution of the system and the solution of the associated homogeneous system.
   Reference for Unit 1: Chapter VIII, Sections 1, 2 of Introduction to Linear Algebra, Serge Lang, Springer Verlag and Chapter 4, of Linear Algebra A Geometric Approach, S. Kumaresan, Prentice-Hall of India Private Limited, New Delhi.

Unit II: Determinants (15 Lectures)

1. Definition of determinant as an $n-$linear skew-symmetric function from $\mathbb{R}^n \times \mathbb{R}^n \times \ldots \times \mathbb{R}^n \longrightarrow \mathbb{R}$ such that determinant of $(E^1, E^2, \ldots, E^n)$ is 1, where E^j denotes the j^{th} column of the $n \times n$ identity matrix I_n. Determinant of a matrix as determinant of its column vectors (or row vectors). Determinant as area and volume.

2. Existence and uniqueness of determinant function via permutations, Computation of determinant of $2 \times 2, 3 \times 3$ matrices, diagonal matrices, Basic results on determinants such as $\det(A^t) = \det(A), \det(AB) = \det(A)\det(B)$, Laplace expansion of a determinant, Vandermonde determinant, determinant of upper triangular and lower triangular matrices.

3. Linear dependence and independence of vectors in $\mathbb{R}^n$ using determinants, The existence and uniqueness of the system $AX = B$, where A is an $n \times n$ matrix wither $\det(A) \neq 0$, Cofactors and minors, Adjoint of an $n \times n$ matrix A, Basic results such as $A\text{adj}(A) = \det(A)I_n$. An $n \times n$ real matrix A is invertible if and only if $\det(A) \neq 0, A^{-1} = \dfrac{1}{\det(A)}\text{adj}(A)$ for an invertible matrix A, Cramer's rule.

4. Determinant as area and volume.

References for Unit 2: Chapter VI of Linear Algebra A geometric approach, S. Kumaresan, Prentice Hall of India Private Limited, 2001 and Chapter VII Introduction to Linear Algebra, Serge Lang, Springer Verlag.

Unit III: Inner Product Spaces (15 Lectures)

1. Dot product in $\mathbb{R}^n$, Definition of general inner product on a vector space over $\mathbb{R}$. Examples of inner product including the inner product $< f, g >= \int_{-\pi}^{\pi} f(t)g(t)\ dt$ on $C[-\pi, \pi]$, the space of continuous real valued functions on $[-\pi, \pi]$.

2. Norm of a vector in an inner product space. Cauchy-Schwartz inequality, Triangle inequality, Orthogonality of vectors, Pythagoras theorem and geometric applications in $\mathbb{R}^2$, Projections on a line, The projection being the closest approximation, Orthogonal complements of a subspace, Orthogonal complements in $\mathbb{R}^2$ and $\mathbb{R}^3$. Orthogonal sets and orthonormal sets in an inner product space, Orthogonal and orthonormal bases. Gram-Schmidt orthogonalization process, Simple examples in $\mathbb{R}^3, \mathbb{R}^4$.

Reference of Unit 3: Chapter VI, Sections 1,2 of Introduction to Linear Algebra, Serge Lang, Springer Verlag and Chapter 5, of Linear Algebra A Geometric Approach, S. Kumaresan, Prentice-Hall of India Private Limited, New Delhi.

Recommended Books:

1. Serge Lang: Introduction to Linear Algebra, Springer Verlag.
2. S. Kumaresan: Linear Algebra A geometric approach, Prentice Hall of India Private Limited.

Additional Reference Books:

1. M. Artin: Algebra, Prentice Hall of India Private Limited.
2. K. Hoffman and R. Kunze: Linear Algebra, Tata McGraw-Hill, New Delhi.
3. Gilbert Strang: Linear Algebra and its applications, International Student Edition.
4. L. Smith: Linear Algebra, Springer Verlag.
5. A. Ramachandra Rao and P. Bhima Sankaran: Linear Algebra, Tata McGraw-Hill, New Delhi.
6. T. Banchoff and J. Wermer: Linear Algebra through Geometry, Springer Verlag Newyork, 1984.
7. Sheldon Axler: Linear Algebra done right, Springer Verlag, Newyork.
8. Klaus Janich: Linear Algebra.
9. Otto Bretcher: Linear Algebra with Applications, Pearson Education.
10. Gareth Williams: Linear Algebra with Applications, Narosa Publication.

USMT 303: Discrete Mathematics

Unit I: Permutations and Recurrence relation (15 lectures)

1. Permutation of objects, S_n, composition of permutations, results such as every permutation is a product of disjoint cycles, every cycle is a product of transpositions, even and odd permutation, rank and signature of a permutation, cardinality of S_n, A_n
2. Recurrence Relations, definition of non-homogeneous, non-homogeneous, linear , non-linear recurrence relation, obtaining recurrence relation in counting problems, solving homogeneous as well as non homogeneous recurrence relations by using iterative methods, solving a homogeneous recurrence relation of second degree using algebraic method proving the necessary result.

Recommended Books:

1. Norman Biggs: Discrete Mathematics, Oxford University Press.
2. Richard Brualdi: Introductory Combinatorics, John Wiley and sons.
3. V. Krishnamurthy: Combinatorics-Theory and Applications, Affiliated East West Press.
4. Discrete Mathematics and its Applications, Tata McGraw Hills.
5. Schaum's outline series: Discrete mathematics,
6. Applied Combinatorics: Allen Tucker, John Wiley and Sons.

Unit II: Preliminary Counting (15 Lectures)

1. Finite and infinite sets, countable and uncountable sets examples such as $\mathbb{N}, \mathbb{Z}, \mathbb{N} \times \mathbb{N}, \mathbb{Q}, (0,1), \mathbb{R}$
2. Addition and multiplication Principle, counting sets of pairs, two ways counting.
3. Stirling numbers of second kind. Simple recursion formulae satisfied by $S(n,k)$ for $k = 1, 2, \cdots, n-1, n$
4. Pigeonhole principle and its strong form, its applications to geometry, monotonic sequences etc.

Unit III: Advanced Counting (15 Lectures)

1. Binomial and Multinomial Theorem, Pascal identity, examples of standard identities such as the following with emphasis on combinatorial proofs.
   - $\sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$
   - $\sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$
   - $\sum_{i=0}^{k} \binom{k}{i}^2 = \binom{2k}{k}$
   - $\sum_{i=0}^{n} \binom{n}{i} = 2^n$
2. Permutation and combination of sets and multi-sets, circular permutations, emphasis on solving problems.
3. Non-negative and positive solutions of equation $x_1 + x_2 + \cdots + x_k = n$
4. Principal of inclusion and exclusion, its applications, derangements, explicit formula for d_n, deriving formula for Euler's function $\phi(n)$.

USMT P03/UAMTP03 Practicals

Suggested Practicals for USMT 301/UAMT303

1. Sequences in $\mathbb{R}^2$ and $\mathbb{R}^3$, limits and continuity of scalar fields and vector fields, using "definition and otherwise" , iterated limits.
2. Computing directional derivatives, partial derivatives and mean value theorem of scalar fields.
3. Total derivative, gradient, level sets and tangent planes.
4. Chain rule, higher order derivatives and mixed partial derivatives of scalar fields.
5. Taylor's formula, differentiation of a vector field at a point, finding Hessian/Jacobean matrix, Mean Value Inequality.
6. Finding maxima, minima and saddle points, second derivative test for extrema of functions of two variables and method of Lagrange multipliers.
7. Miscellaneous Theoretical Questions based on full paper

Suggested Practicals for USMT302/UAMT302:

1. Rank-Nullity Theorem.
2. System of linear equations.
3. Determinants , calculating determinants of 2×2 matrices, $n\times n$ diagonal, upper triangular matrices using definition and Laplace expansion.
4. Finding inverses of $n \times n$ matrices using adjoint.
5. Inner product spaces, examples. Orthogonal complements in $\mathbb{R}^2$ and $\mathbb{R}^3$.
6. Gram-Schmidt method.
7. Miscellaneous Theoretical Questions based on full paper

Suggested Practicals for USMT 303:

1. Derangement and rank signature of permutation.
2. Recurrence relation.
3. Problems based on counting principles, Two way counting.
4. Stirling numbers of second kind, Pigeon hole principle.
5. Multinomial theorem, identities, permutation and combination of multi-set.
6. Inclusion-Exclusion principle. Euler phi function.
7. Miscellaneous theory quesitons from all units.

SEMESTER IV

USMT 401/UAMT 401: CALCULUS IV

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

Unit I: Riemann Integration (15 Lectures)

Approximation of area, Upper/Lower Riemann sums and properties, Upper/Lower integrals, Definition of Riemann integral on a closed and bounded interval, Criterion of Riemann integrability, if $a < c < b$ then $f \in R[a,b]$, if and only if $f \in R[a,c]$ and $f \in R[c,b]$ and $\int_a^b f = \int_a^c f + \int_c^b f$.

Properties:

(i) $f, g \in R[a,b] \Longrightarrow f + g, \lambda f \in R[a,b]$.

(ii) $\int_a^b (f+g) = \int_a^b f + \int_a^b g$.

(iii) $\int_a^b \lambda f = \lambda \int_a^b f$.

(iv) $f \in R[a,b] \Longrightarrow |f| \in R[a,b]$ and $|\int_a^b f| \leq \int_a^b |f|$,

(v) $f \geq 0, f \in C[a,b] \Longrightarrow f \in R[a,b]$.

(vi) If f is bounded with finite number of discontinuities then $f \in R[a,b]$, generalize this if f is monotone then $f \in R[a,b]$.

Unit II: Indefinite and improper integrals (15 lectures)

Continuity of $F(x) = \int_a^x f(t)\ dt$ where $f \in R[a,b]$, Fundamental theorem of calculus, Mean value theorem, Integration by parts, Leibnitz rule, Improper integrals-type 1 and type 2, Absolute convergence of improper integrals, Comparison tests, Abel's and Dirichlet's tests.

Unit III: Applications (15 lectures)

(1) β and Γ functions and their properties, relationship between β and Γ functions (without proff).

(2) Applications of definite Integras: Area between curves, finding volumes by sicing, volumes of solids of revolution-Disks and Washers, Cylindrical Shells, Lengths of plane curves, Areas of surfaces of revolution.

References:

(1) Calculus Thomas Finney, ninth edition section 5.1, 5.2, 5.3, 5.4, 5.5, 5.6.

(2) R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.

(3) Ajit Kumar, S.Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
(4) T. Apostol, Calculus Vol.2, John Wiley.
(5) K. Stewart, Calculus, Booke/Cole Publishing Co, 1994.
(6) J. E. Marsden, A.J. Tromba and A. Weinstein, Basic multivariable calculus.
(7) Bartle and Sherbet, Real analysis.

USMT 402/ UAMT 402: ALGEBRA IV

Unit I: Groups and Subgroups (15 Lectures)

(a) Definition of a group, abelian group, order of a group, finite and infinite groups. Examples of groups including:

- i) $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$ under addition.
- ii) $\mathbb{Q}^*(= \mathbb{Q} \setminus \{0\}), \mathbb{R}^*(= \mathbb{R} \setminus \{0\}), \mathbb{C}^*(= \mathbb{C} \setminus \{0\}).\mathbb{Q}^+$(= positive rational numbers) under multiplication.
- iii) $\mathbb{Z}_n$, the set of residue classes modulo n under addition.
- iv) $U(n)$, the group of prime residue classes modulo n under multiplication.
- v) The symmetric group S_n.
- vi) The group of symmetries of a plane figure. The Dihedral group D_n as the group of symmetries of a regular polygon of n sides (for $n = 3, 4$).
- vii) Klein 4-group.
- viii) Matrix groups $M_{n\times n}(\mathbb{R})$ under addition of matrices, $GL_n(\mathbb{R})$, the set of invertible real matrices, under multiplication of matrices.
- ix) Examples such as S^1 as subgroup of C, μ_n the subgroup of $n-$th roots of unity.

(b) Properties such as

1) In a group $(G, .)$ the following indices rules are true for all integers n, m.
   - i) $a^n a^m = a^{n+m}$ for all a in G.
   - ii) $(a^n)^m = a^{nm}$ for all a in G.
   - iii) $(ab)^n = a^n b^n$ for all ab in G whenever $ab = ba$.
2) In a group $(G, .)$ the following are true:
   - i) The identity element e of G is unique.
   - ii) The inverse of every element in G is unique.
   - iii) $(a^{-1})^{-1} = a$ for all a in G.
   - iv) $(a.b)^{-1} = b^{-1}a^{-1}$ for all a, b in G.
   - v) If $a^2 = e$ for every a in G then $(G, .)$ is an abelian group.
   - vi) $(aba^{-1})^n = ab^n a^{-1}$ for every a, b in G and for every integer n.
   - vii) If $(a.b)^2 = a^2.b^2$ for every a, b in G then $(G, .)$ is an abelian group.
   - viii) $(\mathbb{Z}_n^*, .)$ is a group if and only if n is a prime.
3) Properties of order of an element such as: (n and m are integers.)
   - i) If $o(a) = n$ then $a^m = e$ if and only if n/m.
   - ii) If $o(a) = nm$ then $o(a^n) = m$.
   - iii) If $o(a) = n$ then $o(a^m) = \dfrac{n}{(n, m)}$,. where (n, m) is the GCD of n and m.

iv) $o(aba^{-1}) = o(b)$ and $o(ab) = o(ba)$.

v) If $o(a) = m$ and $o(b) = m, ab = ba, (n, m) = 1$ then $o(ab) = nm$.

(c) Subgroups

i) Definition, necessary and sufficient condition for a non-empty set to be a Subgroup.

ii) The center $Z(G)$ of a group is a subgroup.

iii) Intersection of two (or a family of) subgroups is a subgroup.

iv) Union of two subgroups is not a subgroup in general. Union of two subgroups is a subgroup if and only if one is contained in the other.

v) If H and K are subgroups of a group G then HK is a subgroup of G if and only if $HK = KH$.

Reference for Unit I:

(1) I.N. Herstein, Topics in Algebra.

(2) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Unit II: Cyclic groups and cyclic subgroups (15 Lectures)

(a) Cyclic subgroup of a group, cyclic groups, (examples including $\mathbb{Z}, \mathbb{Z}_n$ and μ_n).

(b) Properties such as:

(i) Every cyclic group is abelian.

(ii) Finite cyclic groups, infinite cyclic groups and their generators.

(iii) A finite cyclic group has a unique subgroup for each divisor of the order of the group.

(iv) Subgroup of a cyclic group is cyclic.

(v) In a finite group $G, G =< a >$ if and only if $o(G) = o(a)$.

(vi) If $G =< a >$ and $o(a) = n$ then $G =< a^m >$ if and only if $(n, m) = 1$.

(vii) If G is a cyclic group of order p^n and $H < G, K < G$ then prove that either $H \subseteq K$ or $K \subseteq H$.

References for Unit II:

(1) I.N. Herstein, Topics in Algebra.

(2) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Unit III: Lagrange's Theorem and Group homomorphism (15 Lectures)

(a) Definition of Coset and properties such as :

1) IF H is a subgroup of a group G and $x \in G$ then

(i) $xH = H$ if and only if $x \in H$.

(ii) $Hx = H$ if and only if $x \in H$.

2) If H is a subgroup of a group G and $x, y \in G$ then

(i) $xH = yH$ if and only if $x^{-1}y \in H$.

(ii) $Hx = Hy$ if and only if $xy^{-1} \in H$.

3) Lagrange's theorem and consequences such as Fermat's Little theorem, Euler's theorem and if a group G has no nontrivial subgroups then order of G is a prime and G is Cyclic.

(b) Group homomorphisms and isomorphisms, automorphisms

i) Definition.

ii) Kernel and image of a group homomorphism.

iii) Examples including inner automorphism.

Properties such as:

(1) $f : G \longrightarrow G'$ is a group homomorphism then $\ker f < G$.

(2) $f : G \longrightarrow G'$ is a group homomorphism then $\ker f = \{e\}$ if and only if f is 1-1.

(3) $f : G \longrightarrow G'$ is a group homomorphism then

(i) G is abelian if and only if G' is abelian.

(ii) G is cyclic if and only if G' is cyclic.

Reference for Unit III:

1. I.N. Herstein, Topics in Algebra.
2. P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Recommended Books:

1. I.N. Herstein, Topics in Algebra, Wiley Eastern Limied, Second edition.
2. N.S. Gopalkrishnan, University Algebra, Wiley Eastern Limited.
3. M. Artin, Algebra, Prentice Hall of India, New Delhi.
4. P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.
5. J.B. Fraleigh, A first course in Abstract Algebra, Third edition, Narosa, New Delhi.
6. J. Gallian. Contemporary Abstract Algebra. Narosa, New Delhi.
7. COmbinatroial Techniques by Sharad S. Sane, Hindustan Book Agency.

Additional Reference Books:

1. S. Adhikari. An introduction to Commutative Algebra and Number theory. Narosa Publishing House.
2. T. W. Hungerford. Algebra, Springer.
3. D. Dummit, R. Foote. Abstract Algebra, John Wiley & Sons, Inc.
4. I.S. Luther, I.B.S. Passi. Algebra. Vol. I and II.

USMT 403: ORDINARY DIFFERENTIAL EQUATIONS

Unit I: First order First degree Differential equations (15 Lectures)

(1) Definition of a differential equation, order, degree, ordinary differential equation and partial differential equation, linear and non linear ODE.

(2) Existence and Uniqueness Theorem for the solution of a second order initial value problem (statement only), Definition of Lipschitz function, Examples based on verifying the conditions of existence and uniqueness theorem

(3) Review of Solution of homogeneous and non-homogeneous differential equations of first order and first degree. Notion of partial derivatives. Exact Equations: General solution of Exact equations of first order and first degree. Necessary and sufficient condition for $Mdx + Ndy = 0$ to be exact. Non-exact equations: Rules for finding integrating factors (without proof) for non exact equations, such as :

  i) $\dfrac{1}{M\ x + N\ y}$ is an I.F. if $M\ x + N\ y \neq 0$ and $Mdx + Ndy = 0$ is homogeneous.

  ii) $\dfrac{1}{M\ x - N\ y}$ is an I.F. if $M\ x - N\ y \neq 0$ and $Mdx + Ndy = 0$ is of the form $f_1(x, y)\ y\ dx + f_2(x, y)\ x\ dy = 0$.

  iii) $e^{\int f(x)\ dx}$ (resp $e^{\int g(y)\ dy}$) is an I.F. if $N \neq 0$ (resp $M \neq 0$) and $\dfrac{1}{N}\left(\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}\right)$ $\left(\text{resp}\dfrac{1}{M}\left(\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}\right)\right)$ is a function of x (resp y) alone, say $f(x)$ (resp $g(y)$).

  iv) Linear and reducible linear equations of first order, finding solutions of first order differential equations of the type for applications to orthogonal trajectories, population growth, and finding the current at a given time.

Unit II: Second order Linear Differential equations (15 Lectures)

1. Homogeneous and non-homogeneous second order linear differentiable equations: The space of solutions of the homogeneous equation as a vector space. Wronskian and linear independence of the solutions. The general solution of homogeneous differential equations. The general solution of a non-homogeneous second order equation. Complementary functions and particular integrals.

2. The homogeneous equation with constant coefficients. auxiliary equation. The general solution corresponding to real and distinct roots, real and equal roots and complex roots of the auxiliary equation.

3. Non-homogeneous equations: The method of undetermined coefficients. The method of variation of parameters.

Unit III: Linear System of ODEs (15 Lectures)

Existence and uniqueness theorems to be stated clearly when needed in the sequel. Study of homogeneous linear system of ODEs in two variables: Let $a_1(t), a_2(t), b_1(t), b_2(t)$ be continuous real valued functions defined on $[a, b]$. Fix $t_0 \in [a, b]$. Then there exists a unique solution $x = x(t), y = y(t)$ valid throughout $[a, b]$ of the following system:

$$\frac{dx}{dt} = a_1(t)x + b_1(t)y,$$
$$\frac{dy}{dt} = a_2(t)x + b_2(t)y$$

satisfying the initial conditions $x(t_0) = x_0 \& y(t_0) = y_0$.
The Wronskian $W(t)$ of two solutions of a homogeneous linear system of ODEs in two variables, result: W (t) is identically zero or nowhere zero on [a, b]. Two linearly independent solutions and the general solution of a homogeneous linear system of ODEs in two variables.
Explicit solutions of Homogeneous linear systems with constant coefficients in two variables, examples.

Recommended Text Books for Unit I and II:

1. G. F. Simmons, Differential equations with applications and historical notes, McGraw Hill.

2. E. A. Coddington, An introduction to ordinary differential equations, Dover Books.

Recommended Text Book for Unit III:
G. F. Simmons, Differential equations with applications and historical notes, McGraw Hill.

USMT P04/UAMT P04 Practicals.

Suggested Practicals for USMT401/UAMT401:

1. Calculation of upper sum, lower sum and Riemann integral.
2. Problems on properties of Riemann integral.
3. Problems on fundamental theorem of calculus, mean value theorems, integration by parts, Leibnitz rule.
4. Convergence of improper integrals, applications of comparison tests, Abel's and Dirichlet's tests, and functions.
5. Beta Gamma Functions
6. Problems on area, volume, length.
7. Miscellaneous Theoretical Questions based on full paper.

Suggested Practicals for USMT402/UAMT 402:

1. Examples and properties of groups.
2. Group of symmetry of equilateral triangle, rectangle, square.
3. Subgroups.
4. Cyclic groups, cyclic subgroups, finding generators of every subgroup of a cyclic group.
5. Left and right cosets of a subgroup, Lagrange's Theorem.

6. Group homomorphisms, isomorphisms.

7. Miscellaneous Theoretical questions based on full paper.

Suggested Practicals for USMT403:

1. Solving exact and non exact equations.

2. Linear and reducible to linear equations, applications to orthogonal trajectories, population growth, and finding the current at a given time.

3. Finding general solution of homogeneous and non-homogeneous equations, use of known solutions to find the general solution of homogeneous equations.

4. Solving equations using method of undetermined coefficients and method of variation of parameters.

5. Solving second order linear ODEs

6. Solving a system of first order linear ODES.

7. Miscellaneous Theoretical questions from all units.

Scheme of Examination

I. **Semester End Theory Examinations:** There will be a Semester-end external Theory examination of 100 marks for each of the courses USMT301/UAMT301, USMT302/UAMT302, USMT303 of Semester III and USMT401/UAMT401, USMT402/UAMT402, USMT403 of semester IV to be conducted by the University.

   1. Duration: The examinations shall be of 3 Hours duration.
   2. Theory Question Paper Pattern:
      a) There shall be FIVE questions. The first question Q1 shall be of objective type for 20 marks based on the entire syllabus. The next three questions Q2, Q2, Q3 shall be of 20 marks, each based on the units I, II, III respectively. The fifth question Q5 shall be of 20 marks based on the entire syllabus.
      b) All the questions shall be compulsory. The questions Q2, Q3, Q4, Q5 shall have internal choices within the questions. Including the choices, the marks for each question shall be 30-32.
      c) The questions Q2, Q3, Q4, Q5 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.
      d) The question Q1 may be subdivided into 10 sub-questions of 2 marks each.

II. **Semester End Examinations Practicals:**
At the end of the Semesters III and IV, Practical examinations of three hours duration and 150 marks shall be conducted for the courses USMTP03, USMTP04.

At the end of the Semesters III and IV, Practical examinations of three hours duration and 150 marks shall be conducted for the courses UAMTP03, UAMTP04.

In semester III, the Practical examinations for USMT301/UAMT301 and USMT302/UAMT302 are held together by the college. The Practical examination for USMT303 is held **separately** by the college.

In semester IV, the Practical examinations for USMT401/UAMT401 and USMT402/UAMT402 are held together by the college. The Practical examination for USMT403 is held **separately** by the college.

Paper pattern: The question paper shall have three parts A, B, C.
Each part shall have two Sections.

Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions. ($8 \times 3 = 24$ Marks)

Section II Problems: Attempt any Two out of Three. ($8 \times 2 = 16$ Marks)

Practical Course	Part A	Part B	**Part C**	Marks out of	duration
USMTP03	Questions from USMT301	Questions from USMT302	Questions from USMT303	120	3 hours
UAMTP03	Questions from UAMT301	Questions from UAMT302	—	80	2 hours
USMTP04	Questions from USMT401	Questions from USMT402	Questions from USMT403	120	3 hours
UAMTP03	Questions from UAMT401	Questions from UAMT402	—	80	2 hours

Marks for Journals and Viva:
For each course USMT301/UAMT301, USMT302/UAMT302, USMT303, USMT401/UAMT401, USMT402/UAMT402 and USMT403:

1. Journals: 5 marks.

2. Viva: 5 marks.

Each Practical of every course of Semester III and IV shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the journal. A student must have a certified journal before appearing for the practical examination.

University of Mumbai

No. AAMS(UG)/ 35 of 2022-23

CIRCULAR:-

Attention of the Principals of the Affiliated Colleges, Directors of the Recognized Institutions in Faculty of Humanities is invited to this office circular No. UG/122 of 2017-18 dated 28th July, 2017 relating to T.Y.B.A./T.Y.B.Sc. Mathematics (Sem V and VI).

They are hereby informed that the recommendations made by the Board of Studies in **Mathematics** at its meeting held on 9th May, 2022 and subsequently passed in the Faculty and then by the Board of Deans at its meeting held on 17th May, 2022 vide item No. 6.1(R) have been accepted by the Academic Council at its meeting held on 17th May, 2022 **vide** item No. 6.9(R) and that in accordance therewith, the revised syllabus of T.Y.B.Sc./B.A. (Mathematics) (Sem V and VI) (CBCS), has been brought into force with effect from the academic year 2022-23. (The same is available on the University's website www.mu.ac.in).

MUMBAI – 400 032
16th June, 2022

(Dr. Vinod Patil)
I/c Director

To

The Principals of the Affiliated Colleges, and Directors of the Recognized Institutions in Faculty of Science & Technology/ Faculty of Humanities.

A.C/6.9/17/05/2022

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No. AAMS(UG)/ 35 -A of 2022-23 16th June, 2022

Copy forwarded with Compliments for information to:-

1) The Dean, Faculty of Science & Technology,
2) The Dean, Faculty of Humanities,
3) The Chairman, Board of Studies Mathematics,
4) The Director, Board of Examinations and Evaluation,
5) The Director, Board of Students Development,
6) The Director, Department of Information & Communication Technology,
7) The Co-ordinator, MKCL.

(Dr. Vinod Patil)
I/c Director

AC – 17/05/2022
Item No. 6.9 (R)

UNIVERSITY OF MUMBAI

Revised Syllabus for T.Y.B. Sc./B.A.

(Mathematics)

Sem – V & VI
(Choice Based Credit System)

(With effect from the academic year 2022-23)

Syllabus for Approval

Sr. No.	Heading	Particulars
1	Title of the Course O. ____________	**T.Y.B. Sc./B.A. (Mathematics)**
2	Eligibility for Admission O. ____________	As per University Regulations
3	Passing Marks R. ____________	40% (Internal 25 (10) Marks and External 75 (30) Marks)
4	Ordinances / Regulations (if any)	
5	No. of Years / Semesters R. ____________	Three Years (Six Semester) Programme
6	Level	~~P.G.~~ / U.G./ ~~Diploma~~ / ~~Certificate~~ **(Strike out which is not applicable)**
7	Pattern	~~Yearly~~ / Semester **(Strike out which is not applicable)**
8	Status	Revised / ~~New /Amended~~ **(Strike out which is not applicable)**
9	To be implemented from Academic Year	From Academic Year 2022-23

Signature
Chairman,
Board of Studies,

Dr Anuradha Majumdar
Dean,
Faculty of Science and Technology

Dean (Science and Technology)

Prof. Anuradha Majumdar (Dean, Science and Technology)

Prof. Shivram Garje (Associate Dean, Science)

Chairperson Board of Studies of Mathematics

Prof. Vinayak Kulkarni

Members of the Board of Studies of Mathematics

Prof. R. M. Pawale

Prof. P. Veeramani

Prof. S. R. Ghorpade

Prof. Ajit Diwan

Dr. S. Aggarwal

Dr. Amul Desai

Dr. S. A. Shende

Dr. Shridhar Pawar

Dr. Sanjeevani Gharge

Dr. Abhaya Chitre

Dr. Mittu Bhattacharya

Dr. Sushil Kulkarni

Dr. Rajiv Sapre

CONTENTS

1. Preamble

The University of Mumbai has brought into force the revised syllabi as per the Choice Based Credit System (CBCS) for the Third year B. Sc / B. A. Programme in Mathematics from the academic year 2022-2023. Mathematics has been fundamental to the development of science and technology. In recent decades, the extent of application of Mathematics to real world problems has increased by leaps and bounds. Taking into consideration the rapid changes in science and technology and new approaches in different areas of mathematics and related subjects like Physics, Statistics and Computer Sciences, the board of studies in Mathematics with concern of teachers of Mathematics from different colleges affiliated to University of Mumbai has prepared the syllabus of T.Y.B. Sc. / T. Y. B. A. Mathematics. The present syllabi of T. Y. B. Sc. for Semester V and Semester VI has been designed as per U. G. C. Model curriculum so that the students learn Mathematics needed for these branches, learn basic concepts of Mathematics and are exposed to rigorous methods gently and slowly. The syllabi of T. Y. B. Sc. / T. Y. B. A. would consist of two semesters and each semester would comprise of four courses and two practical courses for T. Y. B. Sc / T.Y.B.A. Mathematics.

2. Aims and Objectives:

(i) Give the students a sufficient knowledge of fundamental principles, methods and a clear perception of innumerous power of mathematical ideas and tools and know how to use them by modeling, solving and interpreting.

(ii) Reflecting the broad nature of the subject and developing mathematical tools for continuing further study in various fields of science.

(iii) Enhancing students' overall development and to equip them with mathematical modeling abilities, problem solving skills, creative talent and power of communication necessary for various kinds of employment.

(iv) A student should get adequate exposure to global and local concerns that explore them many aspects of Mathematical Sciences.

3. Programme Outcomes:

(i) Enabling students to develop positive attitude towards mathematics as an interesting and valuable subject

(ii) Enhancing students overall development and to equip them with mathematical modeling, abilities, problem solving skills, creative talent and power of communication.

(iii) Acquire good knowledge and understanding in advanced areas of mathematics and physics.

4. Course outcomes:

(i) **Multivariable Calculus II (Sem V):** In this course students will learn the basic ideas, tools and techniques of integral calculus and use them to solve problems from real-life applications including science and engineering problems involving areas, volumes, centroid, Moments of mass and center of mass Moments of inertia. Examine vector fields and define and evaluate line integrals using the Fundamental Theorem of Line Integrals and Green's Theorem; compute arc length.

(ii) **Complex Analysis (Sem VI):** Students Analyze sequences and series of analytic functions and types of convergence, Students will also be able to evaluate complex contour integrals directly and by the fundamental theorem, apply the Cauchy integral theorem in its various versions, and the Cauchy integral formula, they will also be able to represent functions as Taylor, power and Laurent series, classify singularities and poles, find residues and evaluate complex integrals using the residue theorem.

(iii) **Group Theory, Ring Theory (Sem V, Sem VI)** Students will have a working knowledge of important mathematical concepts in abstract algebra such as definition of a group, order of a finite group and order of an element, rings, Euclidean domain, Principal ideal domain and Unique factorization domain. Students will also understand the connection and transition between previously studied mathematics and more advanced mathematics. The students will actively participate in the transition of important concepts such homomorphisms & isomorphisms from discrete mathematics to advanced abstract mathematics.

(iv) **Topology of metric spaces (Sem V), Topology of metric spaces and real analysis (Sem VI):**
This course introduces students to the idea of metric spaces. It extends the ideas of open sets, closed sets and continuity to the more general setting of metric spaces along with concepts such as compactness and connectedness. Convergence concepts of sequences and series of functions, power series are also dealt with. Formal proofs are given a lot of emphasis in this course. This course serves as a foundation to advanced courses in analysis. Apart from understanding the concepts introduced, the treatment of this course will enable the learner to explain their reasoning about analysis with clarity and rigour.

(v) **Partial Differential equations (Sem V: Paper IV: Elective A):**

a. Students will able to understand the various analytical methods for solving first order partial differential equations.

b. Students will able to understand the classification of first order partial differential equations.

c. Students will able to grasp the linear and non linear partial differential equations.

(vi) **Integral Transforms (Sem VI: Paper IV- Elective A):**

a. Students will able to understand the concept of integral transforms and their corresponding inversion techniques.

b. Students will able to understand the various applications of integral transforms.

(vii) **Number Theory and its applications I and II (Sem V, Sem VI):**
The student will be able to

a. Identify and apply various properties of and relating to the integers including primes, unique factorization, the division algorithm, and greatest common divisors.

b. Understand the concept of a congruence and use various results related to congruences including the Chinese Remainder Theorem. Investigate Pseudo-primes , Carmichael number, primitive roots.

c. Identify how number theory is related to and used in cryptography. Learn to encrypt and decrypt a message using character ciphers. Learn to encrypt and decrypt a message using Public-Key cryptology.

d. Express a rational number as a finite continued fraction and hence solve a linear diophantine equation. Express a given repeated continued fraction in terms of a surd. Expand a surd as an infinite continued fraction and hence find a convergent which is an approximation to the given surd to a given degree of accuracy .Solve a Pell equation from a continued fraction expansion

e. Solve certain types of Diophantine equations. Represent a Primitive Pythagorean Triples with a unique pair of relatively prime integers.

f. Identify certain number theoretic functions and their properties. Investigate perfect numbers and Mersenne prime numbers and their connection. Explore the use of arithmetical functions, the Mobius function, and the Euler function.

(viii) **Graph Theory (Sem V: Paper IV- Elective C)**
Upon successful completion of Graph Theory course, a student will be able to:

a. Demonstrate the knowledge of fundamental concepts in graph theory, including properties and characterization of graphs and trees.
b. Describe knowledgeably special classes of graphs that arise frequently in graph theory
c. Describe the concept of isomorphic graphs and isomorphism invariant properties of graphs
d. Describe and apply the relationship between the properties of a matrix representation of a graph and the structure of the underlying graph
e. Demonstrate different types of algorithms including Dijkstra's, BFS, DFS, MST and Huffman coding.
f. Understand the concept of Eulerian graphs and Hamiltonian graphs.
g. Describe real-world applications of graph theory.

(ix) **Graph Theory and Combinatorics (Sem VI: Paper IV -Elective C)**

a. Understand and apply the basic concepts of graph theory, including colouring of graph, to find chromatic number and chromatic polynomials for graphs
b. Understand the concept of vertex connectivity, edge connectivity in graphs and Whitney's theorem on 2-vertex connected graphs.
c. Derive some properties of planarity and Euler's formula, develop the under-standing of Geometric duals in Planar Graphs
d. Know the applications of graph theory to network flows theory.
e. Understand different applications of system of distinct representative and matching theory.
f. Use permutations and combinations to solve counting problems with sets and multisets.
g. Set up and solve a linear recurrence relation and apply the inclusion/exclusion principle.
h. Compute a generating function and apply them to combinatorial problems.

(x) **Basic concepts of probability and random variables (Sem V: Paper IV: Elective D)**
Students will be able to understand the role of random variables in the statistical analysis and use them to apply in the various probability distributions including Binomial distribution, Poisson distribution and Normal distribution. Moreover students will able to apply the concepts of expectations and moments for the evaluation of various statistical measures

(xi) **Operations research (Sem VI: Paper IV: Elective D)**
Students should able to formulate linear programming problem and apply the graphical and simplex method for their feasible solution. Moreover students should understand various alternative operation research techniques for the feasible solution of LPP.

(5) Course structure with minimum credits and Lectures/ Week

SEMESTER V

Course Code	UNIT	TOPICS	Credits	L/Week
Multivariable Calculus II				
USMT 501, UAMT 501	I	Multiple Integrals	2.5	3
	II	Line Integrals		
	III	Surface Integrals		
Group Theory				
USMT 502 ,UAMT 502	I	Groups and Subgroups	2.5	3
	II	Normal subgroups, Direct products and Cayley's theorem		
	III	Cyclic Groups and Cyclic Subgroups Homomorphism		
Topology of Metric Spaces				
USMT 503, UAMT503	I	Metric spaces	2.5	3
	II	Sequences and Complete metric spaces		
	III	Compact Spaces		
Partial Differential Equations(Elective A)				
USMT5A4 ,UAMT 5A4	I	First Order Partial Differential Equations.	2.5	3
	II	Compatible system of first order PDE		
	III	Quasi-Linear PDE		
Number Theory and Its applications I (Elective B)				
USMT5B4 ,UAMT 5B4	I	Congruences and Factorization	2.5	3
	II	Diophantine equations and their & solutions		
	III	Primitive Roots and Cryptography		
Graph Theory (Elective C)				
USMT5C4 ,UAMT 5C4	I	Basics of Graphs	2.5	3
	II	Trees		
	III	Eulerian and Hamiltonian graphs		
Basic Concepts of Probability and Random Variables (Elective D)				
USMT5D4 ,UAMT 5D4	I	Basic Concepts of Probability and Random Variables	2.5	3
	II	Properties of Distribution function, Joint Density function		
	III	Weak Law of Large Numbers		
PRACTICALS				
USMTP05/UAMTP05		Practicals based on USMT501/UAMT 501 and USMT 502/UAMT 502	3	6
USMTP06/UAMTP06		Practicals based on USMT503/ UAMT 503 and USMT5A4/ UAMT 5A4 OR USMT5B4/ UAMT 5B4 OR USMT5C4/ UAMT 5C4 OR USMT5D4/ UAMT 5D4	3	6

SEMESTER VI

BASIC COMPLEX ANALYSIS				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 601, UAMT 601	I	Introduction to Complex Analysis	2.5	3
	II	Cauchy Integral Formula		
	III	Complex power series, Laurent series and isolated singularities		
Ring Theory				
USMT 602 ,UAMT 602	I	Rings	2.5	3
	II	Ideals and special rings		
	III	Factorization		
Topology of Metric Spaces and Real Analysis				
USMT 603 / UAMT 603	I	Continuous functions on Metric spaces	2.5	3
	II	Connected sets		
		Sequences and series of functions		
Integral Transforms(Elective A)				
USMT6A4 ,UAMT 6A4	I	The Laplace Transform	2.5	3
	II	The Fourier Transform		
	III	Applications of Integral Transforms		
Number Theory and Its applications II (Elective B)				
USMT6B4 ,UAMT 6B4	I	Quadratic Reciprocity	2.5	3
	II	Continued Fractions		
	III	Pell's equation, Arithmetic function & and Special numbers		
Graph Theory and Combinatorics (Elective C)				
USMT6C4 ,UAMT 6C4	I	Colourings of Graphs	2.5	3
	II	Planar graph		
	III	Combinatorics		
Operations Research (Elective D)				
USMT6D4 ,UAMT 6D4	I	Basic Concepts of Probability and Linear Programming I	2.5	3
	II	Linear Programming II		
	III	Queuing Systems		
PRACTICALS				
USMTP07/ UAMTP07		Practicals based on USMT601/UAMT 601 and USMT 602/UAMT 602	3	6
USMTP08/UAMTP08		Practicals based on USMT603/ UAMT 603 and USMT6A4/ UAMT 6A4 OR USMT6B4/ UAMT 6B4 OR USMT6C4/ UAMT 6C4 OR USMT6D4/ UAMT 6D4	3	6

Note:

i . USMT501/UAMT501, USMT502/UAMT502, USMT503/UAMT503 are compulsory courses for Semester V.

ii . Candidate has to opt one Elective Course from USMT5A4/UAMT5A4, USMT5B4/UAMT5B4, USMT5C4/UAMT5C4 and USMT5D4/UAMT5D4 for Semester V.

iii . USMT601/UAMT601, USMT602/UAMT602, USMT603/UAMT603 are compulsory courses for Semester VI.

iv . Candidate has to opt one Elective Course from USMT6A4/UAMT6A4, USMT6B4/UAMT6B4, USMT6C4/UAMT6C4 and USMT6D4/UAMT6D4 for Semester VI.

v . Passing in theory and practical and internal exam shall be separate.

(6) Teaching Pattern for T.Y.B.Sc/B.A.

i. Three lectures per week per course (1 lecture/period is of 48 minutes duration).

ii. One practical of three periods per week per course (1 lecture/period is of 48 minutes duration).

(7)Consolidated Syllabus for semester V & VI

SEMESTER V
MULTIVARIABLE CALCULUS II
Course Code: USMT501/UAMT501

ALL Results have to be done with proof unless otherwise stated.

Unit I: Multiple Integrals (15L)

Definition of double (resp: triple) integral of a function and bounded on a rectangle (resp:box). Geometric interpretation as area and volume. Fubini's Theorem over rectangles and any closed bounded sets, Iterated Integrals. Following basic properties of double and triple integrals proved using the Fubini's theorem:

(1) Integrability of the sums, scalar multiples, products, and (under suitable conditions) quotients of integrable functions. Formulae for the integrals of sums and scalar multiples of integrable functions.

(2) Integrability of continuous functions. More generally, Integrability of functions with a "small" set of (Here, the notion of "small sets" should include finite unions of graphs of continuous functions.)

(3) Domain additivity of the integral. Integrability and the integral over arbitrary bounded domains. Change of variables formula (Statement only).Polar, cylindrical and spherical coordinates, and integration using these coordinates. Differentiation under the integral sign. Applications to finding the center of gravity and moments of inertia.

Unit 2: Line Integrals (15L)

Review of Scalar and Vector fields on $\mathbb{R}^n$, Vector Differential Operators, Gradient, Curl, Divergence.
Paths (parametrized curves) in $\mathbb{R}^n$ (emphasis on $\mathbb{R}^2$ and $\mathbb{R}^3$), Smooth and piecewise smooth paths. Closed paths. Equivalence and orientation preserving equivalence of paths. Definition of the line integral of a vector field over a piecewise smooth path. Basic properties of line integrals including linearity, path-additivity and behaviour under a change of parameters. Examples.
Line integrals of the gradient vector field, Fundamental Theorem of Calculus for Line Integrals, Necessary and sufficient conditions for a vector field to be conservative. Green's Theorem (proof in the case of rectangular domains). Applications to evaluation of line integrals.

Unit 3: Surface Integrals (15 L)

Parameterized surfaces. Smoothly equivalent parameterizations. Area of such surfaces.
Definition of surface integrals of scalar-valued functions as well as of vector fields defined on a surface. Curl and divergence of a vector field. Elementary identities involving gradient, curl and divergence. Stoke's Theorem (proof assuming the general from of Green's Theorem). Examples.
Gauss' Divergence Theorem (proof only in the case of cubical domains). Examples.
Reference Books:

1. Apostol, Calculus, Vol. 2, Second Ed., John Wiley, New York, 1969 Section 1.1 to 11.8

2. James Stewart, Calculus with early transcendental Functions - Section 16.5 to 16.9

3. Marsden and Jerrold E. Tromba, Vector Calculus, Fourth Ed., W.H. Freeman and Co., New York, 1996 Section 6.2 to 6.4.

Other References :

1. T. Apostol, Mathematical Analysis, Second Ed., Narosa, New Delhi. 1947.
2. R. Courant and F.John, Introduction to Calculus and Analysis, Vol.2, Springer Verlag, New York, 1989.
3. W. Fleming, Functions of Several Variables, Second Ed., Springer-Verlag, New York, 1977.
4. M. H. Protter and C.B.Morrey Jr., Intermediate Calculus, Second Ed., Springer-Verlag, New York, 1995.
5. G. B. Thomas and R.L Finney, Calculus and Analytic Geometry, Ninth Ed. (ISE Reprint), Addison- Wesley, Reading Mass, 1998.
6. D. V. Widder, Advanced Calculus, Second Ed., Dover Pub., New York. 1989.

Course: Group Theory
Course Code: USMT502/UAMT502

Unit 1: Groups and Subgroups (15L)

(1) Definition and elementary properties of a group. Order of a group. Subgroups. Criterion for a subset to be a subgroup. Abelian groups. Center of a group. Homomorphisms and isomorphisms.

(2) Examples of groups including $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$, Klein 4-group, symmetric and alternating groups, S^1(= the unit circle in $\mathbb{C}$), $\mathrm{GL}_n(\mathbb{R}), \mathrm{SL}_n(\mathbb{R}), \mathrm{O}_n$ (= the group of $n \times n$ nonsingular upper triangular matrices), B_n (= the group of $n \times n$ nonsingular upper triangular matrices), and groups of symmetries of plane figures.

(3) Order of an element. Subgroup generated by a subset of the group.

Unit 2: Normal subgroups, Direct products and Cayley's Theorem (15L)

(1) Cosets of a subgroup in a group. Lagrange's Theorem. Normal subgroups. Alternating group A_n. Listing normal subgroups of A_4, S_3. Quotient (or Factor) groups. Fundamental Theorem of homomorphisms of groups.

(2) External direct products of groups. Examples. Relation with internal products such as HK of subgroups H, K of a group.

(3) Cayley's Theorem for finite groups.

Unit 3: Cyclic groups and cyclic subgroups (15L)

(1) Examples of cyclic groups such as $\mathbb{Z}$ and the group μ_n of the $n-$th roots of unity. Properties of cyclic groups and cyclic subgroups.

(2) Finite cyclic groups, infinite cyclic groups and their generators. Properties of generators.

(3) The group $\mathbb{Z}/n\mathbb{Z}$ of residue classes (mod n). Characterization of cyclic groups (as being isomorphic to $\mathbb{Z}$ or $\mathbb{Z}/n\mathbb{Z}$ for some $n \in \mathbb{N}$).

Recommended Books.

1. I. N. Herstein, Topics in Algebra, Wiley Eastern Limied, Second edition.
2. P. B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.
3. N. S. Gopalkrishnan, University Algebra, Wiley Eastern Limited.
4. M. Artin, Algebra, Prentice Hall of India, New Delhi.
5. J. B. Fraleigh, A first course in Abstract Algebra, Third edition, Narosa, New Delhi.
6. J. Gallian. Contemporary Abstract Algebra. Narosa, New Delhi.

Additional Reference Books

1. T. W. Hungerford. Algebra, Springer.
2. D. Dummit, R. Foote. Abstract Algebra, John Wiley & Sons, Inc.
3. I. S. Luther, I.B.S. Passi. Algebra. Vol. I and II.

Course: Topology of Metric Spaces
Course Code: USMT503/UAMT503

Unit I: Metric spaces (15 L)
Definition and examples of metric spaces such as $\mathbb{R}, \mathbb{R}^2$, $\mathbb{R}^n$ with its Euclidean, sup and sum metrics. $\mathbb{C}$ (complex numbers). l^1 and l^2 spaces of sequences. $C[a, b]$ the space of real valued continuous functions on $[a, b]$. Discrete metric space. Metric induced by the norm. Translation invariance of the metric induced by the norm. Metric subspaces. Product of two metric spaces. Open balls and open sets in a metric space. Examples of open sets in various metric spaces. Hausdorff property. Interior of a set. Properties of open sets. Structure of an open set in $\mathbb{R}$. Equivalent metrics.
Distance of a point from a set, Distance between sets. Diameter of a set. Bounded sets. Closed balls. Closed sets. Examples. Limit point of a set. Isolated point. Closure of a set. Boundary of a set.

Unit II: Sequences and Complete metric spaces (15L)
Sequences in a metric space. Convergent sequence in metric space. Cauchy sequence in a metric space. Subsequences. Examples of convergent and Cauchy sequences in different metric spaces. Characterization of limit points and closure points in terms of sequences. Definition and examples of relative openness/closeness in subspaces. Dense subsets in a metric space and Separability. Definition of complete metric spaces. Examples of complete metric spaces. Completeness property in subspaces. Nested Interval theorem in $\mathbb{R}$. Cantor's Intersection Theorem. Applications of Cantors Intersection Theorem:

(i) The set of real Numbers is uncountable.

(ii) Density of rational Numbers.

(iii) Intermediate Value theorem.

Unit III: Compact spaces (15L)

Definition of a compact metric space using open cover. Examples of compact sets in different metric spaces such as $\mathbb{R}, \mathbb{R}^2, \mathbb{R}^n$ with Euclidean metric. Properties of compact sets: A compact set is closed and bounded, (Converse is not true). Every infinite bounded subset of compact metric space has a limit point. A closed subset of a compact set is compact. Union and Intersection of Compact sets.
Equivalent statements for compact sets in $\mathbb{R}$ with usual metric:

(i) Sequentially compactness property.

(ii) Heine-Borel property.

(iii) Closed and boundedness property.

(iv) Bolzano-Weierstrass property.

Reference books:

1. S. Kumaresan; Topology of Metric spaces.
2. E. T. Copson; Metric Spaces; Universal Book Stall, New Delhi, 1996.
3. P. K. Jain, K. Ahmed; Metric Spaces; Narosa, New Delhi, 1996.

Other references :

1. T. Apostol; Mathematical Analysis, Second edition, Narosa, New Delhi, 1974
2. R. R. Goldberg; Methods of Real Analysis; Oxford and IBH Pub. Co., New Delhi 1970.
3. D. Gopal, A. Deshmukh, A. S. Ranadive and S. Yadav; An Introduction to Metric Spaces, Chapman and Hall/CRC, New York, 2020.
4. W. Rudin; Principles of Mathematical Analysis; Third Ed, McGraw-Hill, Auckland, 1976.
5. D. Somasundaram; B. Choudhary; A first Course in Mathematical Analysis. Narosa, New Delhi
6. G. F. Simmons; Introduction to Topology and Modern Analysis; McGraw-Hi, New York, 1963.
7. Expository articles of MTTS programme.

Course: Partial Differential Equations (Elective A)
Course Code: USMT5A4/UAMT5A4

Unit I: First Order Partial Differential Equations. (15L)

Curves and Surfaces, Genesis of first order PDE, Classification of first order PDE, Classification of integrals, The Cauchy problem, Linear Equation of first order, Lagrange's equation, Pfaffian differential equations. (Ref Book: An Elementary Course in Partial Differential Equations by T. Amaranath, 2nd edition, Chapter 1: 1.1, 1.2, 1.3, Lemma 1.3.1, 1.3.2, 1.3.3, 1.4, Theorem

1.4.1, 1.4.2, 1.5, Theorem 1.5.1, Lemma 1.5.1, Theorem 1.5.2, Lemma 1.5.2 and related examples)

Unit II: Compatible system of first order Partial Differential Equations. (15L)

Definition, Necessary and sufficient condition for integrability, Charpit's method, Some standard types, Jacobi's method, The Cauchy problem. (Ref Book: An Elementary Course in Partial Differential Equations by T. Amaranath, 2nd edition, Chapter 1: 1.6, Theorem 1.6.1, 1.7, 1.8 Theorem 1.8.1, 1.9 and related examples)

Unit III: Quasi-Linear Partial Differential Equations. (15L)

Semi linear equations, Quasi-linear equations, first order quasi-linear PDE, Initial value problem for quasi-linear equation, Non linear first order PDE, Monge cone, Analytic expression for Monge's cone, Characteristics strip, Initial strip. (Ref Book: An Elementary Course in Partial Differential Equations by T. Amaranath, 2nd edition, Chapter 1: 1.10, Theorem 1.10.1, 1.11, Theorem 1.11.1, Preposition 1.11.1, 1.11.2 and related examples)

Reference Books

1. T. Amaranath; An Elementary Course in Partial Differential Equations; 2nd edition, Narosa Publishing house.

2. Ian Sneddon; Elements of Partial Differential Equations; McGraw Hill book.

3. Ravi P. Agarwal and Donal O'Regan; Ordinary and Partial Differential Equations; Springer, First Edition (2009).

4. W. E. Williams; Partial Differential Equations; Clarendon Press, Oxford, (1980).

5. K. Sankara Rao; Introduction to Partial Differential Equations; Third Edition, PHI.

Course: Number Theory and its applications I (Elective B)
Course Code: USMT5B4 / UAMT5B4

Unit I: Congruences and Factorization (15L)

Review of Divisibility, Primes and The fundamental theorem of Arithmetic.
Congruences : Definition and elementary properties, Complete residue system modulo m, Reduced residue system modulo m , Euler's function and its properties, Fermat's little Theorem, Euler's generalization of Fermat's little Theorem, Wilson's theorem, Linear congruence, The Chinese remainder Theorem, Congruences of Higher degree,

Unit II: Diophantine equations and their solutions (15L)

The linear equations $ax + by = c$. The equations $x^2 + y^2 = p$, where p is a prime. The equation $x^2 + y^2 = z^2$, Pythagorean triples, primitive solutions, The equations $x^4 + y^4 = z^2$ and $x^4 + y^4 = z^4$ have no solutions $(x; y; z)$ with $xyz \neq 0$. Every positive integer n can be expressed as sum of squares of four integers, Universal quadratic forms $x^2 + y^2 + z^2 + t^2$. Assorted examples

:section 5.4 of Number theory by Niven- Zuckermann-Montgomery.

Unit III: Primitive Roots and Cryptography (15L)

Order of an integer and Primitive Roots. Basic notions such as encryption (enciphering) and decryption (deciphering), Cryptosystems, symmetric key cryptography, Simple examples such as shift cipher, Affine cipher, Hill cipher, Vigenere cipher. Concept of Public Key Cryptosystem; RSA Algorithm. An application of Primitive Roots to Cryptography.

Reference Books:

1. Niven, H. Zuckerman and H. Montogomery; An Introduction to the Theory of Numbers; John Wiley & Sons. Inc.
2. David M. Burton; An Introduction to the Theory of Numbers; Tata McGrawHillll Edition.
3. G. H. Hardy and E.M. Wright; An Introduction to the Theory of Numbers; Low priced edition; The English Language Book Society and Oxford University Press, 1981.
4. Neville Robins. Beginning Number Theory; Narosa Publications.
5. S.D. Adhikari; An introduction to Commutative Algebra and Number Theory; Narosa Publishing House.
6. N. Koblitz; A course in Number theory and Cryptography; Springer.
7. M. Artin; Algebra; Prentice Hall.
8. K. Ireland, M. Rosen; A classical introduction to Modern Number Theory; Second edition, Springer Verlag.
9. William Stalling; Cryptology and network security.

Course: Graph Theory (Elective C)
Course Code: USMT5C4/UAMT5C4

Unit I: Basics of Graphs (15L)

Definition of general graph, Directed and undirected graph, Simple and multiple graph, Types of graphs- Complete graph, Null graph, Complementary graphs, Regular graphs Sub graph of a graph, Vertex and Edge induced sub graphs, Spanning sub graphs. Basic terminology- degree of a vertex, Minimum and maximum degree, Walk, Trail, Circuit, Path, Cycle. Handshaking theorem and its applications, Isomorphism between the graphs and consequences of isomorphism between the graphs, Self complementary graphs, Connected graphs, Connected components. Matrices associated with the graphs – Adjacency and Incidence matrix of a graph- properties, Bipartite graphs and characterization in terms of cycle lengths. Degree sequence and Havel-Hakimi theorem, Distance in a graph- shortest path problems, Dijkstra's algorithm.

Unit II: Trees (15L)

Cut edges and cut vertices and relevant results, Characterization of cut edge, Definition of a tree and its characterizations, Spanning tree, Recurrence relation of spanning trees and Cayley formula for spanning trees of K_n , Algorithms for spanning tree-BFS and DFS, Binary and m-ary tree, Prefix codes and Huffman coding, Weighted graphs and minimal spanning trees - Kruskal's algorithm for minimal spanning trees.

Unit III: Eulerian and Hamiltonian graphs (15L)

Eulerian graph and its characterization- Fleury's Algorithm-(Chinese postman problem), Hamiltonian graph, Necessary condition for Hamiltonian graphs using $G \setminus S$ where S is a proper subset of $V(G)$, Sufficient condition for Hamiltonian graphs- Ore's theorem and Dirac's theorem, Hamiltonian closure of a graph, Cube graphs and properties like regular, bipartite, Connected and Hamiltonian nature of cube graph, Line graph of graph and simple results.

Reference Books:

1. Bondy and Murty; Grapgh Theory with Applications.
2. Balkrishnan and Ranganathan; Graph theory and applications.
3. Douglas B. West, Introduction to Graph Theory, 2nd Ed., Pearson, 2000

Additional Reference Book:

1. Behzad and Chartrand; Graph theory.
2. Choudam S. A.; Introductory Graph theory.

Course: Basic Concepts of Probability and Random Variables (Elective D)
Course Code: USMT5D4 / UAMT5D4

Unit I: Basic Concepts of Probability and Random Variables.(15 L)

Basic Concepts: Algebra of events including countable unions and intersections, Sigma field $\mathcal{F}$, Probability measure$P$ on $\mathcal{F}$, Probability Space as a triple $(\Omega, \mathcal{F}, P)$, Properties of P including Subadditivity. Discrete Probability Space, Independence and Conditional Probability, Theorem of Total Probability. Random Variable on $(\Omega, \mathcal{F}, P)$ – Definition as a measurable function, Classification of random variables - Discrete Random variable, Probability function, Distribution function, Density function and Probability measure on Borel subsets of $\mathbb{R}$, Absolutely continuous random variable. Function of a random variable; Result on a random variable R with distribution function F to be absolutely continuous, Assume F is continuous everywhere and has a continuous derivative at all points except possibly at finite number of points, Result on density function f_2 of R_2 where $R_2 = g(R_1), h_j$ is inverse of g over a 'suitable' subinterval $f_2(y) + \sum_{i=1}^{n} f_1(h_j(y))|h_j'(y)|$ under suitable conditions.

Reference for Unit 1, Sections 1.1-1.6, 2.1-2.5 of Basic Probability theory by Robert Ash, Dover Publication, 2008.

Unit II: Properties of Distribution function, Joint Density function (15L)

Properties of distribution function F, F is non-decreasing, $\lim_{x \longrightarrow \infty} F(x) = 1$, $\lim_{x \longrightarrow -\infty} F(x) = 0$, Right continuity of F, $\lim_{x \longrightarrow x_0} F(x) = P(\{R < x_o\}, P(\{R = x_o\}) = F(x_o) - F(\overline{x_0})$. Joint distribution, Joint Density, Results on Relationship between Joint and Individual densities, Related result for Independent random variables. Examples of distributions like Binomial, Poisson and Normal distribution. Expectation and $k-$th moments of a random variable with properties.

Reference for Unit II:
Sections 2.5-2.7, 2.9, 3.2-3.3,3.6 of Basic Probability theory by Robert Ash, Dover Publication, 2008.

Unit III: Weak Law of Large Numbers

Joint Moments, Joint Central Moments, Schwarz Inequality, Bounds on Correlation Coefficient ρ ,Result on ρ as a measure of linear dependence, $\text{Var}\Big(\sum_{i=1}^{n} R_i\Big) = \sum_{i=1}^{n} Var(R_i) + 2 \sum_{i=1 \leq i<j \leq n}^{n} \text{Cov}(R_i, R_j)$, Method of Indicators to find expectation of a random variable, Chebyshev's Inequality, Weak law of Large numbers.

Reference for Unit III
Sections 3.4, 3.5, 3.7, 4.1-4.4 of Basic Probability theory by Robert Ash, Dover Publication, 2008.

Additional Reference Books. Marek Capinski, Probability through Problems, Springer.

Course: Practicals (Based on USMT501 / UAMT501 and USMT502 / UAMT502)
Course Code: USMTP05 / UAMTP05

Suggested Practicals (Based on USMT501 / UAMT501)

1. Evaluation of double and triple integrals.
2. Change of variables in double and triple integrals and applications
3. Line integrals of scalar and vector fields
4. Green's theorem, conservative field and applications
5. Evaluation of surface integrals
6. Stoke's and Gauss divergence theorem
7. Miscellaneous theory questions on units 1, 2 and 3.

Suggested Practicals (Based on USMT502 / UAMT502)

1. Examples of groups and groups of symmetries of equilateral triangle, square and rectangle.
2. Examples of determining centers of different groups. Examples of subgroups of various groups and orders of elements in a group.
3. Left and right cosets of a group and Lagrange's theorem.
4. Normal subgroups and quotient groups. Direct products of groups.
5. Finite cyclic groups and their generators

6. Infinite cyclic groups and their properties.
7. Miscellaneous Theory Questions

Course: Practicals (Based on USMT503 / UAMT503 and USMT5A4 OR USMT5B4 OR USMT5C4 OR USMT5D4)
Course Code: USMTP06 / UAMTP06

Suggested Practicals USMT503 / UAMT503:

1. Examples of Metric Spaces, Normed Linear Spaces,
2. Sketching of Open Balls in $\mathbb{R}^2$, Open and Closed sets, Equivalent Metrics
3. Subspaces, Interior points, Limit Points, Dense Sets and Separability, Diameter of a set, Closure.
4. Limit Points ,Sequences , Bounded , Convergent and Cauchy Sequences in a Metric Space.
5. Complete Metric Spaces and Applications.
6. Examples of Compact Sets.
7. Miscellaneous Theory Questions.

Suggested Practicals on USMT5A4/UAMT5A4

1. Find general solution of Langrange's equation.
2. Show that Pfaffian differential equation are exact and find corresponding integrals.
3. Find complete integral of first order PDE using Charpit's Method.
4. Find complete integral using Jacobi's Method.
5. Solve initial value problem for quasi-linear PDE.
6. Find the integral surface by the method of characteristics.
7. Miscellaneous Theory Questions.

Suggested Practicals based on USMT5B4/UAMT5B4

1. Congruences.
2. Linear congruences and congruences of Higher degree.
3. Linear diophantine equation.
4. Pythagorean triples and sum of squares.
5. Cryptosystems (Private Key).
6. Cryptosystems (Public Key) and primitive roots.
7. Miscellaneous theoretical questions based on full USMT5B4 .

Suggested Practicals based on USMT5C4/UAMT5C4

1. Handshaking Lemma and Isomorphism.
2. Degree sequence and Dijkstra's algorithm
3. Trees, Cayley Formula
4. Applications of Trees
5. Eulerian Graphs.
6. Hamiltonian Graphs.
7. Miscellaneous Problems.

Suggested Practicals based on USMT5D4/UAMT5D4

1. Basic concepts of Probability (Algebra of events, Probability space, Probability measure, combinatorial problems)
2. Conditional Probability, Random variable (Independence of events. Definition, Classification and function of a random variable)
3. Distribution function, Joint Density function.
4. Expectation of a random variable, Normal distribution.
5. Method of Indicators, Weak law of large numbers.
6. Conditional density, Conditional expectation.
7. Miscellaneous Theoretical questions based on full paper.

SEMESTER VI
BASIC COMPLEX ANALYSIS
Course Code: USMT601/UAMT601

Unit I: Introduction to Complex Analysis (15 L)
Review of complex numbers: Complex plane, polar coordinates, exponential map, powers and roots of complex numbers, De Moivre's formula, $\mathbb{C}$ as a metric space, bounded and unbounded sets, point at infinity-extended complex plane, sketching of set in complex plane (No questions to be asked).
convergence of sequences of complex numbers and related results. Limit of a function $f : \mathbb{C} \longrightarrow \mathbb{C}$, real and imaginary part of functions, continuity at a point and algebra of continuous functions. Derivative of $f : \mathbb{C} \longrightarrow \mathbb{C}$, comparison between differentiability in real and complex sense, Cauchy-Riemann equations, sufficient conditions for differentiability, analytic function, if f, g analytic then $f + g, f - g, fg$ and f/g are analytic, chain rule.
Theorem: If $f(z) = 0$ everywhere in a domain D, then $f(z)$ must be constant throughout D.
Harmonic functions and harmonic conjugate.

Unit II: Cauchy Integral Formula (15 L)
Evaluation the line integral $\int f(z)\ dz$ over $|z - z_0| = r$ and Cauchy integral formula.

Taylor's theorem for analytic function. Mobius transformations: definition and examples. Exponential function, its properties. trigonometric functions and hyperbolic functions.

Unit III: Complex power series, Laurent series and isolated singularities. (15 L)
Power series of complex numbers and related results. Radius of convergences, disc of convergence, uniqueness of series representation, examples.
Definition of Laurent series , Definition of isolated singularity, statement (without proof) of existence of Laurent series expansion in neighbourhood of an isolated singularity, type of isolated singularities viz. removable, pole and essential defined using Laurent series expansion, examples Statement of Residue theorem and calculation of residue.

Reference Books:

1. J.W. Brown and R.V. Churchill, Complex analysis and Applications : Sections 18, 19, 20, 21, 23, 24, 25, 28, 33, 34, 47, 48, 53, 54, 55 , Chapter 5, page 231 section 65, define residue of a function at a pole using Theorem in section 66 page 234, Statement of Cauchy's residue theorem on page 225, section 71 and 72 from chapter 7.

Other References:

1. Robert E. Greene and Steven G. Krantz, Function theory of one complex variable
2. T.W. Gamelin, Complex analysis

Course: Ring Theory
Course Code: USMT602 / UAMT602

Unit I. Rings (15L)

(1) Definition and elementary properties of rings (where the definition should include the existence of unity), commutative rings, integral domains and fields. Examples, including $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{Z}/n\mathbb{Z}, \mathbb{C}, M_n(\mathbb{R}), \mathbb{Z}[i], \mathbb{Z}[\sqrt{2}], \mathbb{Z}[\sqrt{-5}], \mathbb{Z}[X], \mathbb{R}[X], \mathbb{C}[X], (\mathbb{Z}/n\mathbb{Z})[X]$.

(2) Units in a ring. The multiplicative group of units in a ring R [and, in particular, the multiplicative group F^* of nonzero elemets of a field F]. Description of the units in $\mathbb{Z}/n\mathbb{Z}$. Results such as: A finite integral domain is a field. $\mathbb{Z}/p\mathbb{Z}$, where p is a prime, as an example of a finite field.

(3) Characteristic of a ring. Examples. Elementary facts such as: the characteristic of an itegral domain is either 0 or a prime number.

(Note: From here on all rings are assumed to be commutative with unity).

Unit II. Ideals and special rings(15L)

(1) Ideals in a ring. Sums and products of ideals. Quotient rings. Examples. Prime ideals and maximal ideals. Characterization of prime ideals and maximal ideals in a commutative ring in terms of their quotient rings. Description of the ideals and the prime ideals in $\mathbb{Z}, \mathbb{R}[X]$ and $\mathbb{C}[X]$.

(2) Homomorphisms and isomorphism of rings. Kernel and the image of a homomorphism. Fundamental Theorem of homomorphism of a ring.

(3) Construction of the quotient field of an integral domain (Emphasis on $\mathbb{Z}, \mathbb{Q}$). A field contains a subfield isomorphic to $\mathbb{Z}/p\mathbb{Z}$ or $\mathbb{Q}$.

(4) Notions of euclidean domain (ED), principal ideal domain (PID). Examples such as $\mathbb{Z}, \mathbb{Z}[i]$, and polynomial rings. Relation between these two notions (ED $\Longrightarrow$ PID).

Unit III. Factorization (15L)

(1) Divisibility in a ring. Irreducible and prime elements. Examples.

(2) Division algorithm in $F[X]$ (where F is a field). Monic polynomials, greatest common divisor of $f(x), g(x) \in F[X]$ (not both 0). Theorem: Given $f(x)$ and $g(x) \neq 0$, in $F[X]$ then their greatest common divisor $d(x) \in F[X]$ exists; moreover, $d(x) = a(x)f(x) + b(x)g(x)$ for some $a(x), b(x) \in F[X]$. Relatively prime polynomials in $F[X]$, irreducible polynomial in $F[X]$. Examples of irreducible polynomials in $(\mathbb{Z}/p\mathbb{Z})[X]$ (p prime), Eisenstein Criterion (without proof).

(3) Notion of unique factorization domain (UFD). Elementary properties. Example of a non-UFD is $\mathbb{Z}[\sqrt{-5}]$ (without proof). Theorem (without proof). Relation between the three notions (ED $\Longrightarrow$ PID $\Longrightarrow$ UFD). Examples such as $\mathbb{Z}[X]$ of UFD that are not PID. Theorem (without proof): If R is a UFD, then $R[X]$ is a UFD.

Reference Books

1. N. Herstein; Topics in Algebra; Wiley Eastern Limited, Second edition.
2. P. B. Bhattacharya, S. K. Jain, and S. R. Nagpaul; Abstract Algebra; Second edition, Foundation Books, New Delhi, 1995.
3. N. S. Gopalakrishnan; University Algebra; Wiley Eastern Limited.
4. M. Artin; Algebra; Prentice Hall of India, New Delhi.
5. J. B. Fraleigh; A First course in Abstract Algebra; Third edition, Narosa, New Delhi.
6. J. Gallian; Contemporary Abstract Algebra; Narosa, New Delhi.

Additional Reference Books:

1. S. Adhikari; An Introduction to Commutative Algebra and Number theory; Narosa Publishing House.
2. T.W. Hungerford; Algebra; Springer.
3. D. Dummit, R. Foote; Abstract Algebra; John Wiley & Sons, Inc.
4. I.S. Luthar, I.B.S. Passi; Algebra; Vol. I and II.
5. U. M. Swamy, A. V. S. N. Murthy; Algebra Abstract and Modern; Pearson.
6. Charles Lanski; Concepts Abstract Algebra; American Mathematical Society.
7. Sen, Ghosh and Mukhopadhyay; Topics in Abstract Algebra; Universities press.

Course: Topology of Metric Spaces and Real Analysis
Course Code: USMT603/ UAMT603

Unit I: Continuous functions on metric spaces (15 L)

Epsilon-delta definition of continuity of a function at a point from one metric space to another. Characterization of continuity at a point in terms of sequences, open sets and closed sets and examples. Algebra of continuous real valued functions on a metric space. Continuity of composite function. Continuous image of compact set is compact, Uniform continuity in a metric space, examples (emphasis on $\mathbb{R}$). Results such as: every continuous functions from a compact metric space is uniformly continuous. Contraction mapping and fixed point theorem. Applications.

Unit II: Connected spaces (15L)

Separated sets- Definition and examples. Connected and disconnected sets. Connected and disconnected metric spaces. Results such as: A subset of $\mathbb{R}$ is connected if and only if it is an interval. A continuous image of a connected set is connected.
Characterization of a connected space, viz. a metric space is connected if and only if every continuous function from X to $\{1, -1\}$ is a constant function. Path connectedness in $\mathbb{R}^n$, definition and examples. A path connected subset of $\mathbb{R}^n$ is connected, convex sets are path connected. Connected components. An example of a connected subset of Rn which is not path connected.

Unit III : Sequence and series of functions(15 lectures)

Sequence of functions - pointwise and uniform convergence of sequences of real- valued functions, examples. Uniform convergence implies pointwise convergence, example to show converse not true, series of functions, convergence of series of functions, Weierstrass M-test (statement only). Examples. Properties of uniform convergence: Continuity of the uniform limit of a sequence of continuous function, conditions under which integral and the derivative of sequence of functions converge to the integral and derivative of uniform limit on a closed and bounded interval (statements only). Examples. Consequences of these properties for series of functions, term by term differentiation and integration(statements only). Power series in $\mathbb{R}$ centered at origin and at some point in $\mathbb{R}$, radius of convergence, region (interval) of convergence, uniform convergence, term by-term differentiation and integration of power series, Examples. Uniqueness of series representation, functions represented by power series, classical functions defined by power series such as exponential, cosine and sine functions, the basic properties of these functions.

Reference books:

1. R. R. Goldberg; Methods of Real Analysis; Oxford and International Book House (IBH) Publishers, New Delhi.

2. S. Kumaresan; Topology of Metric spaces.

3. E. T. Copson; Metric Spaces; Universal Book Stall, New Delhi, 1996.

4. Robert Bartle and Donald R. Sherbert; Introduction to Real Analysis; Second Edition, John Wiley and Sons.

Other references:

1. W. Rudin; Principles of Mathematical Analysis.
2. T. Apostol; Mathematical Analysis; Second edition, Narosa, New Delhi, 1974
3. E. T. Copson; Metric Spaces; Universal Book Stall, New Delhi, 1996.
4. P. K. Jain. K. Ahmed, Metric Spaces. Narosa, New Delhi, 1996.
5. W. Rudin, Principles of Mathematical Analysis; Third Ed, McGraw-Hill, Auckland, 1976.
6. D. Somasundaram, B. Choudhary; A first Course in Mathematical Analysis. Narosa, New Delhi
7. G. F. Simmons; Introduction to Topology and Modern Analysis, McGraw-Hi, New York, 1963.
8. Sutherland. Topology.

Course: Intergral Transforms(Elective A)
Course Code: USMT6A4/ UAMT6A4

Unit I: The Laplace Transform (15L)

Definition of Laplace Transform, theorem, Laplace transforms of some elementary functions, Properties of Laplace transform, LT of derivatives and integrals, Initial and final value theorem, Inverse Laplace Transform, Properties of Inverse Laplace Transform, Convolution Theorem, Inverse LT by partial fraction method, Laplace transform of special functions: Heaviside unit step function, Dirac-delta function and Periodic function.

Unit II: The Fourier Transform

Fourier integral representation, Fourier integral theorem, Fourier Sine & Cosine integral representation, Fourier Sine & Cosine transform pairs, Fourier transform of elementary functions, Properties of Fourier Transform, Convolution Theorem, Parseval's Identity.

Unit III: Applications of Integral Transforms

Relation between the Fourier and Laplace Transform. Application of Laplace transform to evaluation of integrals and solutions of higher order linear ODE. Applications of LT to solution of one dimensional heat equation & wave equation. Application of Fourier transforms to the solution of initial and boundary value problems, Heat conduction in solids (one dimensional problems in infinite & semi infinite domain).

Reference Books:

1. Lokenath Debnath and Dambaru Bhatta, Integral Transforms and their Applications, CRC Press Taylor & Francis.
2. I. N. Sneddon, Use of Integral Transforms, Tata-McGraw Hill.

3. L. Andrews and B. Shivamogg, Integral Transforms for Engineers, Prentice Hall of India.

Course: Number Theory and its applications II (Elective B)
Course Code: USMT6B4/ UAMT6B4

Unit I: Quadratic Reciprocity (15 L)

Quadratic residues and Legendre Symbol, Gauss's Lemma, Theorem on Legendre Symbol $\left(\frac{2}{p}\right)$, the result: If p is an odd prime and a is an odd integer with $(a,p)=1$ then $\left(\frac{a}{p}\right) = (-1)^t$ where $t = \sum_{k=1}^{\frac{p-1}{2}} \left[\frac{ka}{p}\right]$, Quadratic Reciprocity law. Theorem on Legendre Symbol $\left(\frac{3}{p}\right)$. The Jacobi Symbol and law of reciprocity for Jacobi Symbol. Quadratic Congruences with Composite moduli.

Unit II: Continued Fractions (15 L)

Finite continued fractions. Infinite continued fractions and representation of an irrational number by an infinite simple continued fraction, Rational approximations to irrational numbers and order of convergence, Best possible approximations. Periodic continued fractions.

Unit III: Pell's equation, Arithmetic function and Special numbers (15 L)

Pell's equation $x^2 - dy^2 = n$, where d is not a square of an integer. Solutions of Pell's equation (The proofs of convergence theorems to be omitted). Arithmetic functions of number theory: $d(n)$ (or $\tau(n)$,) $\sigma(n), \sigma_k(n), \omega(n)$) and their properties, $\mu(n)$ and the Möbius inversion formula. Special numbers: Fermat numbers, Mersenne numbers, Perfect numbers, Amicable numbers, Pseudo primes, Carmichael numbers.

Reference Books:

1. Niven, H. Zuckerman and H. Montogomery; An Introduction to the Theory of Numbers; John Wiley & Sons. Inc.
2. David M. Burton; An Introduction to the Theory of Numbers; Tata McGraw-Hill Edition.
3. G. H. Hardy and E.M. Wright; An Introduction to the Theory of Numbers; Low priced edition; The English Language Book Society and Oxford University Press, 1981.
4. Neville Robins; Beginning Number Theory; Narosa Publications.
5. S. D. Adhikari; An introduction to Commutative Algebra and Number Theory; Narosa Publishing House
6. N. Koblitz; A course in Number theory and Crytopgraphy. Springer.
7. M. Artin; Algebra. Prentice Hall.
8. K. Ireland, M. Rosen; A classical introduction to Modern Number Theory. Second edition, Springer Verlag.

9. William Stalling; Cryptology and network security.

Course: Graph Theory and Combinatorics (Elective C)
Course Code: USMT6C4 /UAMT6C4

Unit I: Colorings of graph (15L)

Vertex coloring- evaluation of vertex chromatic number of some standard graphs, critical graph. Upper and lower bounds of Vertex chromatic Number- Statement of Brooks theorem. Edge colouring- Evaluation of edge chromatic number of standard graphs such as complete graph, complete bipartite graph, cycle. Statement of Vizing Theorem. Chromatic polynomial of graphs- Recurrence Relation and properties of Chromatic polynomials. Vertex and edge cuts, vertex and edge connectivity and the relation between vertex and edge connectivity. Equality of vertex and edge connectivity of cubic graphs. Whitney's theorem on 2-vertex connected graphs.

Unit II: Planar graph (15L)

Definition of planar graph. Euler formula and its consequences. Non planarity of $K_5; K_(3; 3)$. Dual of a graph. Polyhedran in $\mathbb{R}^3$ and existence of exactly five regular polyhedron- (Platonic solids) Colorabilty of planar graphs- 5 color theorem for planar graphs, statement of 4 color theorem. flows in Networks, and cut in a network- value of a flow and the capacity of cut in a network, relation between flow and cut. Maximal flow and minimal cut in a network and Ford-Fulkerson theorem.

Unit III: Combinatorics (15L)

Applications of Inclusion Exclusion Principle- Rook polynomial, Forbidden position problems. Introduction to partial fractions and Newton's binomial theorem for real power series, series expansion of some standard functions. Forming recurrence relation and getting a generating function. Solving a recurrence relation using ordinary generating functions. System of Distinct Representatives and Hall's theorem of SDR.

Recommended Books.

1. Bondy and Murty; Grapgh Theory with Applications.

2. Balkrishnan and Ranganathan; Graph theory and applications.

3. Douglas B. West, Introduction to Graph Theory, 2nd Ed., Pearson, 2000

4. Richard Brualdi; Introduction to Combinatorics.

Additional Reference Book.

1. Behzad and Chartrand; Graph theory.

2. Choudam S. A.; Introductory Graph theory; 3 Cohen, Combinatorics.

Course: Operations Research (Elective D)
Course Code: USMT6D4 / UAMT6D4

Unit I: Linear Programming-I (15L)

Prerequisites: Vector Space, Linear independence and dependence, Basis, Convex sets, Dimension of polyhedron, Faces.
Formation of LPP, Graphical Method. Theory of the Simplex Method- Standard form of LPP, Feasible solution to basic feasible solution, Improving BFS, Optimality Condition, Unbounded solution, Alternative optima, Correspondence between BFS and extreme points. Simplex Method – Simplex Algorithm, Simplex Tableau.

Unit II: Linear programming-II (15L)
Simplex Method – Case of Degeneracy, Big-M Method, Infeasible solution, Alternate solution, Solution of LPP for unrestricted variable. Transportation Problem: Formation of TP, Concepts of solution, feasible solution, Finding Initial Basic Feasible Solution by North West Corner Method, Matrix Minima Method, Vogel's Approximation Method. Optimal Solution by MODI method, Unbalanced and maximization type of TP.

Unit III: Queuing Systems (15L)
Elements of Queuing Model, Role of Exponential Distribution. Pure Birth and Death Models; Generalized Poisson Queuing Model. Specialized Poisson Queues: Steady- state Measures of Performance, Single Server Models, Multiple Server Models, Self- service Model, Machine-servicing Model.

Reference for Unit III:

1. G. Hadley; Linear Programming; Narosa Publishing, (Chapter 3).
2. G. Hadley; Linear Programming; Narosa Publishing, (Chapter 4 and 9).
3. J. K. Sharma; Operations Research; Theory and Applications, (Chapter 4, 9).
4. J. K. Sharma, Operations Research, Theory and Applications.
5. H. A. Taha, Operations Research, Prentice Hall of India.

Additional Reference Books:

1. Hillier and Lieberman, Introduction to Operations Research.
2. Richard Broson, Schaum Series Book in Operations Research, Tata McGrawHill Publishing Company Ltd.

Course: Practicals (Based on USMT601 / UAMT601 and USMT602 / UAMT602)
Course Code: USMTP07 / UAMTP07
Suggested Practicals (Based on USMT601 / UAMT601):

1. Limit continuity and derivatives of functions of complex variables.
2. Steriographic Projection , Analytic function, finding harmonic conjugate.
3. Contour Integral, Cauchy Integral Formula ,Mobius transformations.

4. Taylors Theorem , Exponential , Trigonometric, Hyperbolic functions.
5. Power Series , Radius of Convergence, Laurents Series.
6. Finding isolated singularities- removable, pole and essential, Cauchy Residue theorem.
7. Miscellaneous theory questions.

Suggested Practicals (Based on USMT602 / UAMT602)

1. Examples of rings (commutative and non-commutative), integral domains and fields
2. Units in various rings. Determining characteristics of rings.
3. Prime Ideals and Maximal Ideals, examples on various rings.
4. Euclidean domains and principal ideal domains (examples and non-examples)
5. Examples if irreducible and prime elements.
6. Applications of division algorithm and Eisenstein's criterion.
7. Miscellaneous Theoretical questions on Unit 1, 2 and 3.

Course: Practicals (Based on USMT603 / UAMT603 and USMT6A4 / UAMT6A4 OR USMT6B4 / UAMT6B4 OR USMT6C4 / UAMT6C4 OR USMT6D4 / UAMT6D4)
Course Code: USMTP08 / UAMTP08

Suggested practicals Based on USMT603 / UAMT603:

1 Continuity in a Metric Spaces

2 Uniform Continuity, Contraction maps, Fixed point theorem

3 Connected Sets , Connected Metric Spaces

4 Path Connectedness, Convex sets, Continuity and Connectedness

5 Pointwise and uniform convergence of sequence functions, properties

6 Point wise and uniform convergence of series of functions and properties

7 Miscellaneous Theory Questions.

Suggested Practicals based on USMT6A4 / UAMT6A4

1 Find the Laplace transform of differential and integral equations.

2 Find the inverse Laplace transform by the partial fraction method.

3 Find the Fourier integral representation of given functions.

4 Find the Fourier Sine / Cosine integral representation of given functions.

5 Solve higher order ODE using Laplace transform.

6 Solve one dimensional heat and wave equation using Laplace transform. Solve initial and boundary value problems using Fourier transform.

7 Miscellaneous Theory Questions.

Suggested Practicals based on USMT6B4 / UAMT6B4

1 Legendre Symbol.

2 Jacobi Symbol and Quadratic congruences with composite moduli.

3 Finite continued fractions.

4 Infinite continued fractions.

5 Pell's equations and Arithmetic functions of number theory.

6 Special Numbers.

7 Miscellaneous Theoretical questions.

Suggested Practicals based on USMT6C4 / UAMT6C4

1 Coloring of Graphs

2 Chromatic polynomials and connectivity.

3 Planar graphs

4 Flow theory.

5 Application of Inclusion Exclusion Principle, rook polynomial. Recurrence relation.

6 Generating function and SDR.

7 Miscellaneous theoretical questions.

Suggested Practicals based on USMT6D4 / UAMT6D4
All practicals to be done manually as well as using software TORA / EXCEL solver.

1 LPP formation, graphical method and simple problems on theory of simplex method

2 LPP Simplex Method

3 Big-M method, special cases of solutions.

4 Transportation Problem

5 Queuing Theory; single server models

6 Queuing Theory; multiple server models

7 Miscellaneous Theory Questions.

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(8) Scheme of Evaluation

Scheme of Examination (75:25)

The performance of the learners shall be evaluated into two parts.

- Internal Assessment of 25 percent marks for each paper.
- Semester End Examination of 75 percent marks for each paper.

I. **Internal Evaluation of 25 Marks:**

T.Y.B.Sc. :

(i) One class Test on unit I of 20 marks of duration one hour to be conducted during Practical session.
Paper pattern of the Test:
Q1: Definitions/ Fill in the blanks/ True or False with Justification (04 Marks).
Q2: Multiple choice 5 questions. (10 Marks: 5×2)
Q3: Attempt any 2 from 3 descriptive questions. (06 marks: 2×3)

(ii) Active participation in routine class: 05 Marks.

OR

Students who are willing to explore topics related to syllabus, dealing with applications historical development or some interesting theorems and their applications can be encouraged to submit a project for 25 marks under the guidance of teachers.

T.Y.B.A. :

(i) One class Test on unit I of 20 marks to be conducted during Tutorial session.
Paper pattern of the Test:
Q1: Definitions/ Fill in the blanks/ True or False with Justification (04 Marks).
Q2: Multiple choice 5 questions. (10 Marks: 5×2)
Q3: Attempt any 2 from 3 descriptive questions. (06 marks: 2×3)

(ii) Journal : 05 Marks.

OR

Students who are willing to explore topics related to syllabus, dealing with applications historical development or some interesting theorems and their applications can be encouraged to submit a project for 25 marks under the guidance of teachers.

II. **Semester End Theory Examinations :** There will be a Semester-end external Theory examination of 75 marks for each of the courses USMT501/UAMT501, USMT502/UAMT502, USMT503 and USMT5A4 OR USMT5B4 OR USMT5C4 OR USMT 5D4 of Semester V and USMT601/UAMT601, USMT602/UAMT602, USMT603 and USMT6A4 OR USMT6B4 OR USMT 6C4 OR USMT 6D4 of semester VI to be conducted by the University.

1. Duration: The examinations shall be of $2\frac{1}{2}$ Hours duration.
2. Theory Question Paper Pattern:

a) There shall be FOUR questions. The first three questions Q1, Q2, Q3 shall be of 20 marks, each based on the units I, II, III respectively. The fourth question Q4 shall be of 15 marks based on the entire syllabus.
b) All the questions shall be compulsory. The questions Q1, Q2, Q3, Q4 shall have internal choices within the questions. Including the choices, the marks for each question shall be 30-32.
c) The questions Q1, Q2, Q3, Q4 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.

III. **Semester End Practical Examinations :**
There shall be a Semester-end practical examinations of three hours duration and 100 marks for each of the courses USMTP05/UAMTP05, USMTP06/UAMTP056 of Semester V and USMTP07/UAMTP07, USMTP08/UAMTP08 of semester VI.

In semester V, the Practical examinations for USMTP05/UAPTP05 and USMTP06/UAMTP06 are conducted by the college.

In semester VI, the Practical examinations for USMTP07/UAMTP07 and USMTP08/UAMTP08 are conducted by the University.
Question Paper pattern:

Paper pattern: The question paper shall have two parts A, B.
Each part shall have two Sections.

Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions. ($8 \times 3 = 24$ Marks)

Section II Problems: Attempt any Two out of Three. ($8 \times 2 = 16$ Marks)

Practical Course	Part A	Part B	Marks out of	duration
USMTP05/UAMTP05	Questions from USMT501/UAMT501	Questions from USMT502/UAMT502	80	3 hours
USMTP06/UAMTP06	Questions from USMT503/UAMT503	Questions from USMT504/UAMT504	80	3 hours
USMTP07/UAMTP07	Questions from USMT601/UAMT601	Questions from USMT602/UAMT602	80	3 hours
USMTP08/UAMTP08	Questions from USMT603/UAMT603	Questions from USMT604/UAMT604	80	3 hours

Marks for Journals and Viva:
For each course USMT501/UAMT501, USMT502/UAMT502, USMT503/UAMT503, USMT504/UAMT504, USMT601/UAMT601, USMT602/UAMT602 USMT603/UAMT603, and USMT604/UAMT604:

1. Journals: 5 marks.
2. Viva: 5 marks.

Each Practical of every course of Semester V and VI shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the certified journal.

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